CAMBRIDGE MATHEMATICAL SERIES.

EXAMPLES IN ALGEBRA.

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IN

ALGEBRA

BY

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NOTE.—*This volume can be had with or without answers.*

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PREFACE.

THESE examples are intended to provide a complete course of elementary algebra for classes in which the bookwork is supplied by the teacher.

In the choice of the subjects included, and in their arrangement, I have throughout followed the recommendations of the Committee on the Teaching of Mathematics appointed by the Mathematical Association.

Among new features may be mentioned:—

The postponement until after easy simultaneous equations of the 'long rules' for multiplication and division.

The postponement until after quadratics of complicated fractions, H.C.F., L.C.M., Square root, and Literal Equations.

The early introduction and extensive use of Graphs.

The inclusion of some of the applications to Geometry which form such a prominent feature in modern continental text-books.

The treatment of fractional indices from a numerical point of view, so as to lead up to the use of four-figure logarithm tables.

The stress laid on numerical checks of all kinds.

The large selection of problems, including very easy ones.

A number of examples have been taken from examination papers recently set at Charterhouse.

It remains for me to acknowledge that I am much indebted to my colleague, Mr. H. Crabtree, to whom are due the whole of the problems in Ex. 65, as well as those in Ex. 98; and also to the Rev. J. R. Colthurst and Mr. A. D. Tuckey, who have given me much kind assistance in checking the answers and in proof-reading.

C. O. TUCKEY.

CONTENTS.

EXAMPLES IN ALGEBRA.

Ex. 1. **USE OF SIGNS.**

(1) Verify that

$$16(3-2)=16.3-16.2,$$
$$14(7+1)=14.7+14.1,$$
$$18(5-3)=18.5-18.3.$$

Remove the brackets in

$$4(a-b),\ 12(a+b),\ a(b-c).$$

(2) Verify that

$$62+3(4-1)=62+3.4-3.1,$$
$$62-3(4-1)=62-3.4+3.1,$$
$$75+3(5-1)=75+3.5-3.1,$$
$$75-3(5-1)=75-3.5+3.1,$$
$$100-2(18+4)=100-2.18-2.4.$$

Remove the brackets in

$$a+3(b-c),$$
$$a-3(b-c),$$
$$a-x(b+c).$$

(3) Work out (i) as in arithmetic, (ii) as in algebra,

$$9(7+5)-9(7-5),$$
$$14(3+2)-14(3-2).$$

Work out $7(a+b)-7(a-b)$.

(4) Find the value of

$$5.53-4.9,$$
$$5.(53-4.9),$$
$$5.(53-4).9.$$

(5) Work out (i) as in arithmetic, (ii) as in algebra,

$$9(9+1)-9^2,$$
$$12(12+3)-12^2.$$

Work out $x(x+a)-x^2$.

(6) Find the difference between

$$4+12\,.\,3-2 \text{ and } (4+12)(3-2);$$

also between $15-3\frac{1}{2}\times 2$ and $(15-3\frac{1}{2})2$;

also between $12-4\frac{1}{3}\times 3$ and $(12-4\frac{1}{3})3$.

By removing the bracket

show that $a-b\,.\,c$ and $(a-b)c$ are different.

(7) Multiply out

$$2^5,\ 3^4,\ 2\,.\,7^2,\ 2\,.\,3\,.\,5^2,\ \left(\frac{1}{2}\right)^3,\ 2^2\,.\,\frac{1}{3}\,.\,9.$$

Write out in full

$$a^5,\ b^7,\ \frac{b^8}{b^3},\ a^4b^3c.$$

(8) Put into prime factors, using the index notation for repeated factors:

186, 121, 56, 64, 66, 625, 343, 72, 100, 275, 48, 286, 63, 1715, 41, 85, 120, 105.

(9) State as identities the results of the following divisions:

$$164\div 2,\ 195\div 3,\ 195\div 15,\ 160\div 3,\ \frac{184}{7},\ \frac{131}{37},\ \frac{300}{4},\ \frac{2521}{29}.$$

Using letters, give the connection between dividend, divisor, quotient, and remainder.

(10) State as a series of identities the work of finding by the 'long rule' the G.C.M. of 163 and 134; also of 183 and 21; also 1491 and 399.

(11) What are $9+0$, 9×0, $9+1$, 9×1?

(12) Write in simpler form if possible

$$x+0,\ x\times 0,\ x+1,\ x\times 1.$$

(13) What must be subtracted from 12 to leave 0; what from y?

(14) By what must 15 be divided to give 1 as answer?

By what must x be divided to give 1 as answer?

Ex. 2. NEGATIVE QUANTITIES.

(1) What is the value of

$$7-4,\ 4-7,\ 8-14,\ 1-12,$$
$$-3-12,\ -3+2-12,\ -4+7-3?$$

Which is greater, the sum of 6 and -2,
or the difference of 3 and -2?

(2) What must be added to the following numbers to make the result 0?

$$-8,\ -11,\ 9,\ 3-4,\ 3-x,\ a-b.$$

(3) Express (i) as a gain, (ii) as a loss, the result of each of the following transactions:

A loss of £10 followed by a gain of £15.

A loss of £5 followed by a gain of 10s.

A gain of £4 followed by a gain of 5s.

(4) Express in symbols the combined weight of a balloon pulling upwards with a force of 500 lbs. and two masses of iron of weights 300 lbs. and 150 lbs.

(5) How many degrees of frost are temperatures of 5°, $-5°$, 15°, $-7°$?

(6) What is the difference between the temperatures of 40° and $-3°$?

(7) Find the average midnight temperature for the week in which the midnight temperatures were

$$30°,\ 18°,\ 10°,\ 4°,\ 0°,\ -7°,\ -20°.$$

(8) Find the average number of degrees of frost at midnight for a week in which the midnight temperatures were

$$34°,\ 31°,\ 27°,\ 24°,\ 28°,\ 30°,\ 32°.$$

(9) A train was due at a station at 10 minutes to 5. How many minutes before 5 did it arrive if it was a quarter of an hour late?

In Figure 1—

(10) What numbers represent OA, OB, OC, OD, OE?

(11) What numbers represent OA, AB, BC, CD, DE?

(12) Illustrate from the figure that

$$7 - 9 = -2,$$
$$9 - 18 + 14 = 5,$$
$$5 - 14 + 7 = -2.$$

In Figure 2—

(13) What numbers represent OA, OB, OC, OD, OE?

(14) What numbers represent OA, AB, BC, CD, DE?

(15) Illustrate from the figure that

$$2 - 5 = -3,$$
$$6 - 9 + 1 = -2$$
$$-7 + 4 + 5 = 2.$$

Draw figures to illustrate that

(16) $7 - 5 = 2$, $5 - 7 = -2$, $5 - 3 - 4 + 2 = 0$.

(17) $5 - 7 + 4 - 3 = -1$, $-2 + 1 + 3 + 3 = +5$, $2 - 4 + 2 = 0$.

In Figure 3—

(18) What pairs of numbers explain the position of the points A, B, C, ... M, starting from Ox, Oy?

(19) What pairs of numbers explain how to get from

(i) A to C; (ii) B to E; (iii) D to G; (iv) K to M;
(v) F to O; (vi) L to M; (vii) I to D; (viii) J to F?

(20) Draw on squared paper two lines to start from (the axes), and mark with letters A, B, C, ... the points given by the following pairs of numbers:

$$(4, 2),\ (2, 4),\ (6, 1),\ (4, 0),\ (-2, 6),\ (2, -6),$$
$$(3, -4),\ (-7, -2),\ (-5, 0),\ (2, -3),\ (-1, -1),\ (-4, 5)$$

Fig.1.

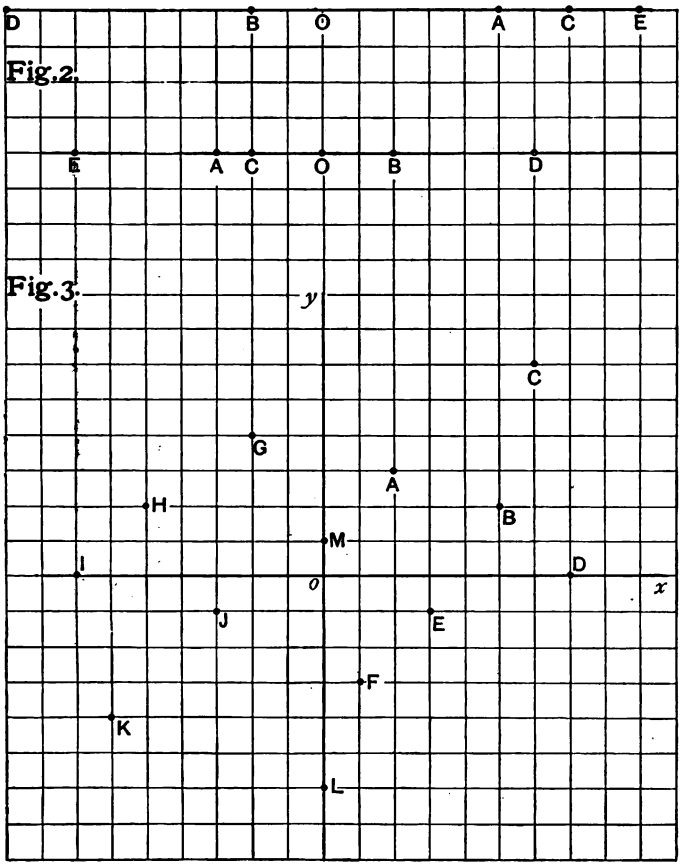

Fig.2.

Fig.3.

Ex. 3. **SUBSTITUTION.**

Oral.

(1) If $a=4$, what is the value of
$2a,\ 3a,\ -a,\ a^2,\ a+3,\ a^2-6$?

(2) If $x=3$, what is the value of
$2x,\ 2x+4,\ x^2,\ x^2-1,\ x^2-x$?

(3) If $a=5$ and $b=2$, what is the value of
$a+b,\ a-b,\ 2a+b,\ a+2b,\ 2a-3b$;
$a^2,\ a^2+b,\ a^2+b^2,\ a^2-b^2,\ a^2-3b^2$;
$a^3,\ b^3,\ a^3-b^3,\ a^3+b^3,\ ab^2$?

(4) If $y=\frac{1}{2}$, what is the value of
$2y,\ 3y,\ 5y,\ y^2,\ 1-y^2,\ \frac{1}{2}-y^2,\ y-y^2$?

(5) If $x=2$ and $y=3$, what is the value of
$x+y,\ 2x-y,\ x^2-y,\ xy$; $x^2y,\ xy^2,\ x^5y$?

(6) If $a=3$ and $b=1$, what is the value of
$a^3,\ b^3,\ ab^2,\ a^2+2b^2,\ 4ab+b^2$?

(7) If $x=-2$, what is the value of
$3x,\ -5x,\ x^2,\ -2x^2,\ x^3,\ 7+2x$?

(8) If $a=1$, $b=-3$, what is the value of
$a+b,\ a-b,\ 2a+5b,\ a^2+b^2,\ ab,\ b^2-2ab$?

(9) If $x=3$, $c=-3$, what is the value of
$x+c,\ x-c,\ c-x,\ cx,\ xc,\ x^2+c^2$?

(10) If $a=-1$, $b=-2$, what is the value of
$4a+b,\ 3a-2b,\ a^2+b^2,\ a^2-b^2,\ 2ab$?

Ex. 4.

(1) If $a=3$, $b=7$, find the value of
(i) $16a-3b$; (ii) $2a+b^2-ab+30$,
(iii) $a^3-2a^2b+ab^2$.

(2) Find the value of the three expressions in question 1, if $a=1$, $b=1$.

(3) If $c=4$, $d=7$, find the value of
(i) $7c-3d+cd$; (ii) $d^2-c^2-3cd+41$;
(iii) c^3-2d^2+8d-4.

(4) Find the value of the three expressions in question 3, if $c=3$, $d=3$.

(5) If $a=1$, $b=5$, $c=0$, find the value of

(i) $3a+4b-6c$; (ii) a^2-b^2+2c;
(iii) $16ab-4bc+2ac$; (iv) $c^2-c(2a+3b)$;
(v) $b^3-4ac+22$; (vi) $3(a+b)-2c(a+c)+a^2(b+c)$.

(6) If $c=6$, $d=2$, $e=0$, find the value of

(i) $3c-4d+2e$; (ii) $c^2-d^2+e^2$;
(iii) $2cd-de+3ec$; (iv) c^3+ed^2-7cd;
(v) $e(c+d)+3e^2$; (vi) $4(c+d)(c-d)-7cde$.

(7) If $x=2$, $y=-3$, find the values of

(i) $(2x+y)(x-y)$; (ii) x^2+y^2+xy;
(iii) $(x+y)(x-y)$; (iv) $3(x+y)-4(x-y)$.

(8) If $a=-1$, $b=2$, find the values of

(i) $2ab+b^2$; (ii) $a^2-2ab+b^2$;
(iii) $(2a-b)(a+2b)$; (iv) a^2b+ab^2.

(9) Find the values of x^2-2x+3 for the following five values of x:

(i) $x=1$; (ii) $x=2$; (iii) $x=5$;
(iv) $x=9$; (v) $x=10$.

(10) For the same five values of x as in question 9, find the values of the expression $(x^2-2)(x-1)$.

(11) Find the values of $y^2+2y-18$ for the following five values of y:

(i) $y=1$; (ii) $y=0$; (iii) $y=7$;
(iv) $y=12$; (v) $y=100$.

(12) For the same five values of y as in question 11, find the values of the expression $(y^2-4)(y+1)$.

(13) Find the values of x^3-x+1 for the following values of x:

(i) $x=2$; (ii) $x=1$; (iii) $x=0$;
(iv) $x=-1$; (v) $x=-2$.

(14) For the same five values of x as in question 13, find the values of $(x+2)x(x+1)$.

Ex. 5. IDENTITIES AND EQUATIONS.

(1) Show that $a^2-(a-3)(a+3)-9$ is 0 if a is 1, 2, 3, or 4.

(2) Show that $b^2-1-(b+1)(b-1)$ is 0 if b has any of the values 2, 4, 10, 0.

(3) Show that a^2-6a+8 is 0 if $a=2$ and also if $a=4$. Find a value of a for which it is not 0.

(4) Show that b^2-4b+3 is 0 if $b=3$ and also if $b=1$. Find a value of b for which it is not 0.

(5) The following statements are identities; show by substitution that each is true if $b=3$ and $c=1$:

(i) $2(b-c)+5c=3(b+c)-b$;

(ii) $b^2-c^2=(b+c)(b-c)$;

(iii) $(b+c)^2=b^2+2bc+c^2$.

(6) Show by substitution that each of the following statements is true if $a=4$, $d=3$, also if $a=5$, $d=-1$.

(i) $3(a+d)+2a=5(a-d)+8d$;

(ii) $(a-d)^2=a^2-2ad+d^2$;

(iii) $a^2-d^2=(a+d)(a-d)$.

(7) If $x=8$ show that $4(x-5)=x+4$.

(8) If $y=3$ prove that $7y-40=5-8y$.

(9) If $z=6$ then $z^2-9(z-2)=0$.

(10) If $x=40$ then $7x-100=20+4x$.

(11) If $x=1$ verify that $2x^3-x+1=4x^2-2$.

(12) Show that if $x=2$ then $x^3+x-10=0$.

(13) Test which of the following equations are satisfied by $x=1$; which by $x=3$; and which by $x=4$:

(i) $5x-2=4x+2$;

(ii) $3x+2=18-3x+2$;

(iii) $x^2-5x+4=0$;

(iv) $x^2+x=12$;

(v) $2x+3(x-1)=4(x+2)+1$;

(vi) $x^3-8x^2+19x=12$;

(vii) $x^3=2x-1$.

(14) Test whether $y=-1$ or $y=-3$ satisfies

(i) $2(y+6)+4=4(2-y)-10$;

(ii) $y^2+4y+3=0$;

(iii) $y^2-4y=3$;

(iv) $y^3+2y^2=y+2$.

(15) Test which of the following equations are satisfied by $x=1$; which by $x=2$; and which by $x=5$:

(i) $7x-10=3(x+3)-8x+5$;

(ii) $4(x-5)+3x-2(x-2)=9$;

(iii) $x^2-3x+2=0$;

(iv) $x^2-7x+10=0$;

(v) $x^3=8x^2-17x+10$;

(vi) $x^2-7x+12=0$;

(vii) $2x^3=7(x+2)-12$.

(16) Test each of the following equations for the solutions $x=0$, $x=1$, $x=-1$:

(i) $x^2-2x+1=0$;

(ii) $x^3=x(x+7)$;

(iii) $x^3+4x^2+2x+6=0$;

(iv) $2x^4+3x(x^2-1)=2x^2$.

Ex. 6. COLLECTING LIKE TERMS.

State the result of collecting like terms and also the number of terms in the original expression and in the answer:

(1) $3a+7a-9a$. (2) $b-3b+4b$. (3) $x-7x+2x$.

(4) $3y-8y+2y$. (5) $3x^2-2x^2+5x^2-4x^2+x^2$.

(6) $19yz+2yz-10yz-5yz-6yz$. (7) $5x+2y-3x$.

(8) $ab-2c+5c-2ab$. (9) $2yz-x-3yz+2x+yz$.

(10) $a^2+2a+3a^2-4a^2+2a+17a-3a^2+12a+4a^2$.

(11) $2x-11+2-2x$. (12) $bx-3bx+5bx+2bx-7bx+3bx+2bx$.

(13) $5x-2x+3x-4x+2\frac{1}{2}x$. (14) $7a-\frac{1}{2}a+\frac{3}{2}a+2a$.

(15) $3abcd-6abcd+43abcd-abcd+2abc$.

(16) $2b-b^2+3b-\frac{1}{2}b+\frac{3}{4}b^2-4$.

(17) $-ab+3ab-6ab-2abc+ab-9ab$.

(18) $-\frac{1}{2}x^2+x^2-\frac{3}{4}x^2+\frac{x^2}{4}-\frac{3}{2}x^2$.

(19) $2x+c^2-3c^2+8x-\frac{1}{2}c^2+cx$.

(20) $3a+2b-c+\frac{1}{2}a-2b+3c-2a+b-1\frac{1}{2}c+6a-15b+8c$.

Ex. 7. ADDITION AND SUBTRACTION.

The answers to be said or written down without preliminary work.

(1) Find the sum of $a+2b$, $-3a+b$, $6a-4b$.

(2) Add together $a+c$, $-5a+3c$, $9a-4c$.

(3) Take c from $2a+3c$.

(4) From $8x-9y$ subtract $3x+2y$.

(5) Add together $6a+b-c$, $3a-b+c$, $2a+2b+2c$.

(6) From 2 take $1-x^2$.

(7) Subtract $13x^2+2x$ from $19x^2+4x+1$.

(8) Add $6x$ to $3x^2$ and from the result take $7x$.

(9) Add $3x^2$ to $13x$ and from the result take 3.

(10) Find the sum of $16ax+a^2x^2$, $3-4ax$, $2a^2x^2$, $3a^2x^2+2ax-8$.

(11) From zero subtract $1-2x^2$ and to the result add 3.

(12) What is the difference between $19x^2$ and $6x^2$?

(13) What is the difference between $19x^2$ and $6x$?

(14) Subtract $3+4x$ from the sum of $9+2x$ and $8-x$.

(15) What is the result of adding together x, y, and z?

(16) Express in symbols the result of diminishing $9b$ by the sum of $3c$ and $4b$.

(17) What is the remainder when $1-4y^2+y^3$ is subtracted from $5y^3$?

(18) Take x^2-y^2 from $16xy-5y^2$.

(19) A boy works $x+y$ sums of which $x-7$ are wrong. How many are right?

(20) To unity add the sum of x^2+x and $x-7$.

(21) What must be added to $3a$ to produce $10a$?

(22) What must be added to x^2+x to produce x^3-x^2+3x?

(23) What must be taken from x to produce $3a$?

(24) What is the term in x^2 in the sum of $3x^2-x$, $2-x+4x^2$, x^3-3x^2, $1-2x+5x^2$?

(25) Take y from x and add to the result $14y$.

(26) What is the remainder if from $7x - y^2$ we take $3x$?

(27) What must be added to -4 to produce 10.

(28) Add 3 to -1.

(29) Subtract b from x.

(30) Subtract $-a$ from $-b$.

(31) Add $x+y$ and $x-y$.

(32) What must be subtracted from x that the result may be $x-y$?

(33) What must be subtracted from x that the result may be $x+y$?

(34) From the sum of $2x+3y$ and $3x-2y$ subtract $x+y$.

(35) If the square of x be doubled, what must be added to it to produce its cube?

(36) How much greater is $3x^2$ than $2x^2-x+7$?

(37) By how much does x^3 exceed x^3-3x+4?

(38) By how much is a^2+b^2 less than $a^2+2ab+b^2$?

(39) To the sum of $a+b$ and $a-b$ add $2b$.

(40) By how much does a^2+3ab^2 exceed $3a^2b+b^3$?

Ex. 8. BRACKETS.

Simplify by removing brackets and collecting like terms:

(1) $2a+3(b+c-a)+7b$.

(2) $a+6(b-c)-2(a-b+x)$.

(3) $4x+x^2-(x^2-x)+2$.

(4) $a+\{b+c-(a+b-c)\}$.

(5) $2a+3b-\{b^2+2bc+x+7b^2-(3bc+2x-b^2)\}$.

(6) Add $2x$ to the sum of $x-(4x+2)$ and $7-(3x-1)$.

(7) Find the sum of

$$x^2-\{x^2+(2x+3)-(4x+2)\} \text{ and } x^2-(x-7).$$

(8) Subtract $x^2-(2x+3)$ from $8x^2+(2-6x)$.

(9) Add together $a+b-c$, $a-(b+c)$, $a-(b-c)$, $-(a+b+c)$.

(10) If P stands for $x-y$ and Q for $x+y$, find the value of $P+Q-(2P-Q)$.

(11) Simplify $6a-\{3b-(2b+b-a)\}$.

(12) Add $a+b$ to $(6a+5b)-\{a-(b-2a)\}$.

(13) If L stands for $a+b$ and M for $a-b$, find the value of $2L-M-(L-2M)$.

(14) Add together

$$x^2-(x-2),\ -x^2+x-2,\ -(x^2-x)+2,\ x^2+(x-2).$$

(15) Simplify

$$x^3-[2x^3+3x-\{x^2+2x+(x-4)-2x^3-(x^2-x)\}+17x^2].$$

Ex. 9. MULTIPLICATION.

State in their simplest forms the results of the following multiplications:

(1) $x^2\times x^4$. (2) $a^3\times a^5$. (3) $b^2x\times b^3$.

(4) $ba^2\times a^3b$. (5) $cx\times x^2ab$. (6) $(-x)\times x$.

(7) $x\times(-x^2)$. (8) $a\times(-b^2)$. (9) $(-a)(-a)$.

(10) $(-a)^2$. (11) $2ax\times(-x)^2$. (12) $x^2\times(-x)$.

(13) $(-x)^2.(-x)$. (14) $(-x)^3$. (15) $(-c)^2$.

(16) $(-a)^4$. (17) $(-b)^5$. (18) $7ax\times 3a^2$.

(19) $\frac{1}{2}bx\times 2x^2$. (20) $a^2\times b^2ac$. (21) $abx\times bcx\times(-acx)$.

(22) $(-6bx)\times 3ax$. (23) $\frac{1}{3}x^2\times(-\frac{1}{2}xy)\times 12x^2y^2$.

(24) $\frac{1}{2}x\times\frac{1}{4}y$. (25) $\frac{1}{3}x\times(-\frac{1}{2}xy)$.

(26) $a^2b\times b^2c\times c^2a$. (27) $(-a)(-b)(-c)$.

(28) $(-a)^2(-b)^3$. (29) $2ab.4xy$. (30) $x^3y.y^3z.z^3x$.

(31) Multiply by 12: $\frac{x}{12}, \frac{x}{4}, \frac{y}{3}, \frac{2x}{3}, \frac{3x}{4}, \frac{5y}{6}$.

(32) Multiply by 20: $\frac{x}{20}, \frac{7a}{20}, \frac{7x}{4}, \frac{x}{5}, \frac{1}{4}y, \frac{3y}{2}$.

Ex. 10. DIVISION.

State in their simplest forms the results of the following divisions:

(1) $x^7\div x^2$. (2) $b^3\div b$. (3) $c^8\div c^3$.

(4) $2ab^2\div a$. (5) $2ab^2\div b$. (6) $7xyz\div 7y$.

(7) $7xyz\div xyz$. (8) $9x^3\div 3x$. (9) $9x^9\div 3x^3$.

(10) $-18xy\div 9$. (11) $-6x^2\div x$. (12) $-x^3y\div xy$.

(13) $x^5y\div(-x)$. (14) $abc^2\div c$. (15) $-abc^2\div(-c)$.

(16) $abc^2\div(-b)$. (17) $x^3z\div(-x^2z)$. (18) $(-a)^5\div a^2$.

(19) $(2a)^3\div 4$. (20) $a^3bc\div(-a)^2$.

Ex. 11.

State in their simplest form with brackets removed:

(1) $x^3(x^2 - 1)$.
(2) $x^7 \times (-3bc)$.
(3) $5x^5 \div x^2$.
(4) $(2a + 4b) \div 2$.
(5) $7(8 - 3x^2)$.
(6) $x(y - z) + x(z - y)$.
(7) $2x^3 \times (a + 3b)$.
(8) $(-3b)^2$.
(9) $(-4x)^2$.
(10) $(-11bc)^3$.
(11) $(bc)^4$.
(12) $(ax^2)^5$.
(13) $6(a - b) + 3b$.
(14) $2(x^2 - y^2) \times ax$.
(15) $3(x^2y - xy^2) \div 3y$.
(16) $(x^2y + xy^2) \div xy$.
(17) $abc \div (-a)$.
(18) $xyz \div (-2x)$.
(19) $3x^2y \div (-x)$.
(20) $2(a + b - c) + 2(c + a - b)$.
(21) $2x^2y \times (-a)^2$.
(22) The product of x and 32.
(23) The sum of x^2 and the square of $(-x)$.
(24) The product of $12x$ and $\frac{1}{2}x$.
(25) The result of multiplying $4x^2y$ by $3xy^3$.
(26) The result of dividing $20x^2$ by 5.
(27) The quotient when $20x^2$ is divided by $5x$.
(28) The number of times that 81 contains 3.
(29) The number of times that $60x^3$ contains x^2.
(30) How often $7x$ is contained in $21ax$.
(31) The result of multiplying by 4: $\frac{x+1}{2}$, $\frac{3(x-1)}{4}$, $\frac{x-3}{8}$.
(32) The result of multiplying by 12:
$\frac{x-1}{6}$, $\frac{3x+2}{4}$, $\frac{1}{2}(x+1)$, $\frac{1}{4}(x-3)$, $\frac{2}{3}x$, $\frac{3}{4}(y+2)$.
(33) The result of multiplying by 20: $\frac{x-8}{20}$, $\frac{7x-15}{20}$, $\frac{3y+1}{4}$.

Ex. 12.

Simplify by removing brackets and collecting like terms:

(1) $a(b - c) + b(c - a) + c(a - b)$.
(2) $7(6a - 2b) + 3(a - 3c) + 2c(7 - a)$.
(3) $a^2(b^2 - c^2) + b^2(c^2 - a^2) + c^2(a^2 - b^2)$.
(4) $a^2(b - c) + b^2(c - a) + c^2(a - b)$.
(5) $a(b + c) + b(c + a) + c(a + b)$.
(6) $x(2y - z) + y(2z - x) + z(2x - y)$.

Simplify by removing brackets and collecting like terms:

(7) $x(a+3b)-a(b+2x)+b(x-a)$.

(8) $p(2q-s+t)+t(p-s)-q(2p-t)$.

(9) $a(b^2-c^2)+b(c^2-a^2)+c(a^2-b^2)+a^2(b-c)+b^2(c-a)+c^2(a-b)$.

(10) $x(x+y)-y(x+y)$. (11) $x(x-y)+y(x-y)$.

If P stands for x^3+3x^2+3x+1, Q for x^2+2x+1, and R for $x+1$:

(12) $P-xQ$. (13) $Q-xR-R$.

(14) $P-Q-x^2R-xR$. (15) $P-xQ-Q$.

If L stands for x^2+y^2, M for x^2-y^2:

(16) Lx^2-My^2. (17) $L+M-x(x+y)$.

(18) $Lx+My+(L+M)(x+y)$.

(19) $a(x-y)+b(y-z)+c(z-x)-x(b-c)-y(c-a)-z(a-b)$.

(20) In the identity $(a+b)m=am+bm$ replace m by

(i) $a+b$, (ii) $a-b$,

(iii) $a+c$, (iv) $c+d$,

and obtain the expanded forms of the four products $(a+b)^2$, $(a+b)(a-b)$, $(a+b)(a+c)$, $(a+b)(c+d)$.

(21) In the identity $(a-b)m=am-bm$ replace m by

(i) $a-b$, (ii) $a-c$,

(iii) $c+d$, (iv) $c-d$,

and obtain the expanded forms of four products.

Ex. 13. SIMPLE EQUATIONS.

Oral.

(1)	$4x=12$.	(2)	$7x=56$.	(3)	$2x=108$.
(4)	$10x=5$.	(5)	$12x=3$.	(6)	$40=5x$.
(7)	$11y=33$.	(8)	$12y=42$.	(9)	$3y=7$.
(10)	$18=-6x$.	(11)	$3x=-96$.	(12)	$14x=0$.
(13)	$12a=24$.	(14)	$6a=-18$.	(15)	$3a=-5$.
(16)	$2x=0$.	(17)	$42=-7y$.	(18)	$33=11a$.
(19)	$-17=4x$.	(20)	$-6y=63$.	(21)	$1=13x$.
(22)	$14x=1$.	(23)	$4x=0$.	(24)	$0=-3x$.
(25)	$26x=39$.	(26)	$7=21x$.	(27)	$6x+3=21$.
(28)	$2x-5=10$.	(29)	$18=3x-3$.	(30)	$5=8x+3$.

(31) $3x = 20 + x$. (32) $-x = 10 + x$. (33) $3y + 4 = 16$.

(34) $2y + 1 = y + 2$. (35) $4x + 7 = x + 7$. (36) $5x - 8 = -3x$.

(37) $\frac{x}{4} = 2$. (38) $\frac{x}{5} = 12$. (39) $\frac{2x}{5} = 8$.

(40) $\frac{x}{3} - 1 = 0$. (41) $x = 6 + \frac{x}{2}$. (42) $x - \frac{x}{2} = 13$.

(43) $\frac{x}{3} + \frac{x}{3} = 10$. (44) $\frac{2x}{3} = \frac{x}{3} + 7$. (45) $\frac{x}{2} = 10 - \frac{x}{2}$.

Ex. 14.

Solve, and in each case check the answer:

(1) $5x + 49 = 4x + 56$. (2) $3x + 10 = 5x - 100$.

(3) $7x - 11 = 2 + 5x - 9$.

(4) $2x - 17 + 9x + 2 - x - 43 + 14x = 26$.

(5) $8 - 3x = 6x + 10 - x + 6$. (6) $4x + 5 = 13 - 3x + 8 + x$.

(7) $0 = 2 - 9x + 53 - 2x$. (8) $18x + 4 = 34x - 4$.

(9) $23x + 40 = 4x + 2$. (10) $4x + 7 = 4 + 2x$.

(11) $14 - 2x - 23 + 13x + 81 = 164 - 5x + 7 + 12x - 3x + 18$.

(12) $4(7 - x) = 2(7x - 31)$. (13) $7(x - 18) = 3(x - 14)$.

(14) $3(3 + x) = 4(7x + 10)$. (15) $5(x + 3) = 7(9 - x)$.

(16) $8(23 + 2x) + 13(x - 3) = 0$. (17) $8 - 5(13 - 2x) = x - 3$.

(18) $8 - 3(19 - 6x) + 7(21x - 9) = 3x$.

(19) $6 - 8(x - 3) + 2(5x + 6) = 0$. (20) $7(x - 3) = 9(x + 1) - 38$.

(21) $\frac{1}{2}x = \frac{x}{3} + 2$. (22) $\frac{x}{2} - \frac{x}{5} = \frac{x}{4} + 1$.

(23) $\frac{1}{2}x = \frac{2x}{3} - 4$ (24) $\frac{x}{8} + \frac{x}{4} + \frac{x}{2} = x - 4$.

(25) $\frac{x}{2} - \frac{x}{3} = \frac{4x}{9} - 15$. (26) $\frac{1}{2}x + \frac{1}{4}x = 1\frac{3}{4} - x$.

(27) $\frac{x}{5} + 2 = 1\frac{1}{2} + \frac{x}{20} - \frac{x}{5}$. (28) $\frac{1}{2}x - \frac{3}{4} + 7x = 3x + 1\frac{1}{2}$.

(29) $\frac{x}{3} - \frac{x}{4} = 2\frac{2}{3}$. (30) $\frac{x}{3} + \frac{x - 8}{4} = 5$.

(31) $\frac{x - 1}{4} + \frac{x + 3}{5} = 8$. (32) $\frac{x + 1}{10} - 3 = 0$.

(33) $9 - \frac{x - 3}{2} = \frac{x + 3}{4}$. (34) $\frac{x + 1}{4} - \frac{x - 1}{5} = 1$.

Solve, and in each case check the answer:

(35) $\frac{2(x+1)}{5} - 8 = \frac{2x}{16} - 1.$ (36) $\frac{1}{2}(x-2) - \frac{1}{4}(x+2) = \frac{1}{3}(x-3).$

(37) $\frac{x+1}{2} - \frac{1}{4} = x - \frac{2x-1}{3}.$ (38) $\frac{x}{7} + \frac{x-1}{2} - x = -1.$

(39) $\frac{x-7}{8} = \frac{x-3}{11}.$ (40) $\frac{x+5}{12} = \frac{1-6x}{5}.$

(41) $6x - 10 = \cdot 2x + 3\cdot 34.$ (42) $3x - 4 = 2(x - \cdot 3) - x.$

(43) $\cdot 5(x+1) - \cdot 3x = 1\cdot 12.$ (44) $\cdot 125x - \cdot 375 = \cdot 06 + \cdot 185(x-3).$

(45) $8(x-3) = 2\cdot 24.$ (46) $2(x-5) = (x+4)3 - 5\cdot 6.$

(47) $5(x+2) = 3\cdot 67.$ (48) $2\cdot 34 = 4(x - 1\cdot 5).$

(49) $3(\cdot 2x - \cdot 5) = 1\cdot 4.$ (50) $2(x-7) = x + 4\cdot 63.$

Ex. 15. ON SYMBOLS.

Oral.

One Number. (Before problems 1—23).

The number being x, what represents:

(1) double it.
(2) half it.
(3) five times the number.
(4) eight times it with three added.
(5) the sum of the number and six.
(6) the product of four and the number.
(7) the result of taking 5 times it from 70.
(8) the excess of the number over 20.
(9) how much the number is less than 43.
(10) the product of 12 and the sum of the number and five?

The number being y, by what are the same 10 things represented.

Two Numbers. (Before problems 24—32.)

(1) Two numbers differ by 8: let x be the smaller, what is (i) the larger, (ii) their sum, (iii) their product, (iv) twice the larger?

(2) The difference of two numbers is 5: let y be the larger what is (i) the smaller, (ii) twice the smaller − the larger, (iii) their sum, (iv) their sum + their difference?

(3) The sum of two numbers is 120: let x be the larger, what is (i) the smaller, (ii) their difference, (iii) their product?

(4) 340 is divided into 2 parts of which z is one, what is (i) the other, (ii) twice the other, (iii) their difference?

Consecutive Numbers: Ages. (Before problems 33—43).

(1) What is the whole number (i) next above x, (ii) next below y?

(2) If x be an even number, what is (i) the next even number, (ii) the next even number but one, (iii) the even number just before?

(3) If z be an odd number, what is (i) the next odd number, (ii) the next number, (iii) the odd number next below z?

(4) If A is now x years old, (i) how old will he be 10 years hence, (ii) how old was he five years ago, (iii) what will be his age in a years, (iv) how long is it since he was six, (v) in how many years' time will he be 100?

(5) If A's age is now x years, B's age y years, (i) what is the sum of their ages, (ii) what was A's age 6 years ago, (iii) what was the sum of their ages 6 years ago, (iv) what is the difference of their ages now, (v) what will be the difference of their ages 7 years hence?

Ex. 16. PROBLEMS.

(1) A certain number is multiplied by 23 and 117 is then added: the result is 232; find the number.

(2) From the double of a number 7 is taken: the result is 95; find the number.

(3) A number is multiplied by 7 and 164 is then added: the result is 395; find the number.

(4) Three times a number is subtracted from 235: the result is 217; find the number.

(5) If 4 times a number is taken from 37 the result is 19; what is the number?

(6) If 3 is added to a number and the result multiplied by 27, the product is 175; find the number.

(7) A number is diminished by 5 and the result multiplied by 7, making 203; find the number.

(8) A number is added to three times itself: the result is 464; find the number.

(9) A number is multiplied by 6 and 14 added: from this the original number is taken: the result is 169; find the number.

(10) Four times a number with 33 added is equal to 7 times the number with 18 added; find the number.

(11) Twice a certain number is as much above 20 as three times the number is below 65; find the number.

(12) The sum of 87 and three times a certain number (to be found) is equal to 5 times the number.

(13) How much must be added to 56 that the resulting number may be eight times the added part?

(14) How much must be taken from 129 that the result may be twice what was taken away?

(15) The excess over 60 of five times the number required is equal to the number.

(16) Eight times a number is as much below 100 as 33 times the number is below 275; find the number.

(17) If 12 be added to a number and the result multiplied by 3, the product is equal to 6 times the result of adding 1 to the number; find the number.

(18) Three times the sum of the required number and 13 is equal to 5 times the sum of the number and 2.

(19) The difference between half the number to be found and one third of it is 11.

(20) Find the number which exceeds its third part by 232.

(21) From three times the required number 36 is taken and the result is half the number.

(22) What is the number whose 7th part exceeds its 8th part by 2?

(23) A number, its half, and its third part make 110; find the number.

(24) One number exceeds another by 7, and their sum is 133; find the numbers.

(25) Find two numbers which differ by 19, and whose sum is 121.

(26) Divide 140 into 2 parts which differ by 22.

(27) Divide 40 into 2 parts which differ by 1.

(28) Divide £257 between A and B so that A gets £9 more than B.

(29) A and B have £157 between them; A has £31 more than B; how much has each?

(30) £100 is distributed between A, B, and C, so that B has £10 more than A, and C has £8 more than B; how much has each?

(31) £233 is divided between A, B, C; A's share is double B's share, and C gets £5 more than B; what is the share of each?

(32) A has £7 more than B and £5 less than C; between them they have £88; how much has each?

(33) The sum of two consecutive numbers is 91; find them.

(34) Two consecutive odd numbers are such that twice the smaller exceeds the greater by 29; find them.

(35) The sum of three consecutive even numbers is 120; find them.

(36) Find two consecutive even numbers such that one is $\frac{10}{11}$ of the other.

(37) A is 35 years old, B 7 years old; in how many years' time will A be twice as old as B?

(38) A father is 10 times as old as his son: six years hence he will be 4 times as old; what are their ages?

(39) A is 35, B is 7, and C is 5 years old; how long will it be before A's age is the sum of the ages of B and C?

(40) A father is 30 years older than his son, who is 16; how long ago was he four times as old as his son?

(41) How old is a man if his age 10 years hence will be double what his age was $10\frac{1}{2}$ years ago?

(42) Half B's age is equal to a third of A's age, A being 16 years older than B; find their ages.

(43) A father is now three times as old as his son was 10 years ago; if he is 34 years older than his son, how old is he?

Ex. 17. FROM WORDS TO SYMBOLS.

(1) State in figures with signs:

Add 3 to 4 and multiply the result by 12.
Subtract 2 from 5 and multiply the result by 7.
From 13 take 8 and double the result.
Multiply by 6 the difference between 77 and 74.
Multiply 2 by $7\frac{1}{2}$ and subtract 4 from the result.
Divide 6 by $3\frac{1}{2}$ and add twice $4\frac{1}{2}$ to the result.

(2) State:

Add 2 to a and multiply the result by b.
From x take 7 and multiply the result by 3.
Double y and then subtract 17.
To half of z add four times x.
Divide 18 by a and add to the result y.
Multiply by x the result of taking b from 100.

(3) State:

the cost of a things at b pence each.
the cost of 1 thing if a things cost b pence.
the cost at 1s. each of a things.
the cost of a things when 5 of them cost c pence.
the cost of a things when x of them cost c pence.
the cost of x things at 1s. per score.

(4) State:

the cost of c things at d shillings each.
the cost of a things at 5 pence per dozen.
the cost of x things at a pence per dozen.
the cost of a things at 1d. each.
the cost of d things at four for a penny.
the cost of y things if x things cost z shillings.

(5) State:

the number of pence in x shillings.
the number of pence in a pounds.
the number of pence in a pounds, b shillings.
the number of shillings in 100 pence.
the number of shillings in x pence.

(6) How far does one go

in 18 hours at 25 miles an hour.
in x hours at 32 miles an hour.
in b hours at c miles an hour.
in 180 minutes at x miles an hour.
in z minutes at x miles an hour.

(7) State (with signs) the simple interest

on £845 at 3 per cent. for 2 years.
on £640 at 4 per cent. for 3 years.
on £450 at x per cent. for 2 years.
on £A at 2 per cent. for b years.
on £P at R per cent. for Y years.

(8) For the rooms (i)–(v) state in symbols

(α) the area of the floor, (β) the perimeter,
(γ) the area of the walls, (δ) the volume.

(i) length 40 feet, breadth 25 feet, height 14 feet.
(ii) „ 36 „ „ 24 „ „ x „
(iii) „ a „ „ 20 „ „ 15 „
(iv) „ l „ „ b „ „ 20 „
(v) „ l „ „ b „ „ h „

Use letters to state briefly the following known results:

(9) The sum of two numbers is equal to their difference plus twice the smaller number.

(10) The sum of two numbers plus their difference is equal to twice the larger number.

(11) Half the sum of two numbers added to half their difference makes up the larger number.

(12) Half the difference of two numbers added to the smaller number makes half their sum.

(13) The seventh part of a number minus its eighth part is equal to its fifty-sixth part.

(14) If one third of any number be added to one quarter of it, the result is the same as if seven times the number were divided by twelve.

(15) When we multiply together two numbers it does not matter in which order we multiply them.

(16) To divide any number by a fraction, we invert the fraction and multiply.

(17) The difference of two numbers multiplied by the larger one is equal to the square of the larger one minus the product of the two numbers.

(18) The sum of two numbers multiplied by one of them is the same as the square of that one plus the product of the two numbers.

(19) If the sum of two numbers be multiplied by a third number the result is the same as if each of the two numbers were multiplied separately by the third number and these results added together.

(20) If the difference of two numbers be multiplied by a third number the result is the same as if each of the two numbers were multiplied separately by the third number and these results subtracted.

Ex. 18. FORMULAE.

(1) Find the circumferences of circles of diameters 1, 2, 3, 4, 5 inches, being given that the number of inches c in the circumference of a circle whose diameter is d inches is found from

$$c = \frac{22}{7} d.$$

(2) Find the number of square feet in the areas of the circles whose radii are 1 ft., 2 ft., 5 ft., 7 ft., 20 ft., 28 ft., being given that the number of square feet A in the area of a circle whose radius is r feet is found from

$$A = \frac{22}{7} r^2.$$

(3) If a stone falls freely, the space s feet through which it has dropped in a time t seconds from starting is given by the formula $s = 16t^2$.

Find s for the values $t = 1$, 2, etc., up to 10, also for the values

$$t = 15 \text{ and } t = \frac{1}{2}.$$

(4) Find the weight in lbs. of a square beam of pine, 1 ft. square and 30 ft. long, from the formula

$$W = 40\, ab^2$$

(where a feet is the length of the beam which is b feet square). Find also the weight of a beam 6 ins. square and 30 ft. long.

(5) Find the weight of a beam of oak, 25 ft. long and 18 ins. square, from the formula

$$W = 52\, ab^2 \text{ [see question 4].}$$

(6) Find the weight in lbs. of a square beam of beech, 2 ft. square and 12 ft. long, from the formula

$$W = 43\, ab^2 \text{ [see question 4].}$$

(7) The weight W lbs. of a cubical block of limestone, the side of the cube being a feet, is given by the formula

$$W = 171\, a^3.$$

Find the weight in lbs. of three cubical blocks whose sides are respectively 1 ft., 1½ ft., and 8 ins.

(8) The weight W lbs. of a cubical block of granite, the side of the cube being a feet, is given by the formula

$$W = 168\,a^3.$$

Find the weight in lbs. of three cubical blocks of granite whose sides are respectively 1 ft., 2 ft., and 6 ins. long.

(9) Find the sum of all the odd numbers less than 100 from the formula

$$\text{sum} = \frac{n}{2}(f+l),$$

where f is the first of the series of numbers,
„ l is the last „ „
„ n is the number „ „

(10) From the formula given in question 9, viz.:

$$\text{sum} = \frac{n}{2}(f+l), \text{ find}$$

(i) the sum of all the odd numbers between 100 and 200;
(ii) the sum of all the even numbers less than 60;
(iii) the sum of the numbers 10, 15, 20, 25..., there being 20 numbers altogether.

[The formula given applies in all cases where the numbers increase or decrease by regular intervals.]

Ex. 19. MISCELLANEOUS.

(1) Add together $a+b-c$, $2a-b+c$, $3c-6b+a$, $a-b-c$.

(2) Show that if $a=2$, $b=1$, $c=-1$,
then $a^3+b^3+c^3-3abc=(a+b+c)(a^2+b^2+c^2-bc-ca-ab)$.

(3) Test whether the equation $x^2-6x+5=0$ is satisfied by $x=1$, or $x=3$, or $x=5$.

(4) Is the equation $x^3=30-x$ satisfied by $x=3$?

(5) From x^3+2x^2-5x+4 take x^2-5x+4.

(6) Solve the equations:

(i) $2x+3=27-4x$;

(ii) $\frac{2}{3}(x+5)=x-\frac{x-10}{4}$.

(7) Subtract $x^3 - 1$ from the sum of $x^3 + 2x^2$ and $3x^2 - x + 2$.

(8) Solve the equation $5x + 2 = 3(x - 4) + 7x - 2(3 - x)$.

(9) Verify that $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$ for the particular case when $a = 7$ and $b = 2$.

(10) Add $a^2 + 2ab + b^2$, $a^2 - 2ab + b^2$, $a^2 + 4ab - b^2$.

(11) What is the cost of:

(i) x things if y of them cost z pence;

(ii) a things at b pence per dozen?

(12) From the formula

$$S = C + \frac{C \cdot R}{100},$$

find C when $S = 51$ and $R = 2$.

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(13) Find the sum of $x^2 - a^2$, $x^2 + 2ax - 5a^2$, $x^2 + 3a^2$, $7ax$.

(14) Take away $a + b + c$ from $a - b - c$.

(15) Show that $x = 5$ satisfies the equation $x^3 - 2x^2 = 12x + 15$.

(16) Add together $\frac{1}{2}a + b$, $a + \frac{3}{4}b$, $-2a - \frac{1}{2}b$, $\frac{3a}{2} - \frac{b}{4}$.

(17) Solve the equations:

(i) $\frac{x}{9} - \frac{x}{11} = 2$; (ii) $3(x - 5) = 4{\cdot}32$.

(18) Find three consecutive numbers whose sum is 84.

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(19) If 23 be added to x^4, find the sum of the result and $x^3 - 2x^4 + 2$.

(20) What is the distance travelled:

(i) in x hrs. at y miles per hour;

(ii) in an hour if a miles takes b mins.?

(21) From the formula

$$S = 4 \cdot \frac{22}{7} \cdot r^2,$$

find the surface S of a sphere whose radius r is $3\frac{1}{2}$ ins.; also of one whose radius is 5 ins.

(22) Solve the equation

$$\frac{x}{2}-\frac{x}{7}+\frac{x}{3}-\frac{x}{6}=x-20.$$

(23) Find the number such that if 18 be added to it, three times the result is equal to nine times the number.

(24) The ages of a father and son are 32 years and 8 years. How soon will the son's age be half that of his father?

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(25) Verify that if $a=3$ and $b=1$ then

$$(a+b)(a^2-ab+b^2)=a^3+b^3.$$

(26) From the formula $s=14t-16t^2$ find s if $t=\frac{1}{2}$, and also if $t=3$.

(27) Add to x^2-3x the sum of $24+2x$ and $3x^2-x+5$.

(28) Subtract $\frac{1}{2}x^2-xy+\frac{2}{3}y^2$ from $x^2-\frac{1}{2}xy+y^2$.

(29) Solve the equations:

(i) $7(x-3)=5(x-1)$;

(ii) $\frac{2x}{3}-\frac{7x}{11}=1$.

(30) Find two consecutive even numbers such that their sum is 45 times their difference.

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(31) Write down the value of

(i) $(-x^2y)^4+2x^3.x^5y^4$; (ii) $(-3ab)^5$.

(32) Add together $\frac{x}{2}+y+\frac{z}{3}$, $\frac{2x}{3}-\frac{y}{2}+\frac{2z}{3}$, $x+y-\frac{z}{2}$.

(33) Simplify $x^3(ax^2+bx-c)-a(x^5+2x^3)-bx^2(x^2+x)$.

(34) A room is half as long again as it is broad. If its breadth had been 5 feet more and its length 4 feet less the room would have been square. Find its length and breadth.

(35) Write down the simple interest on £x at $2\frac{1}{2}$ per cent. for y years.

If the interest is £$\frac{1}{2}x$ find y.

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(36) From $a^3+b^3-6ab^2+14a^2b$ take $-7ab^2-9a^2b+b^3$.

(37) Simplify $x(x-y-2z)+2x(y-z-3x)-x(3y+z)$.

(38) Find two consecutive numbers such that the difference of their squares is 97.

(39) Divide £1100 between A, B, C, so that A gets £2 for each £3 that B gets, and C gets twice as much as A.

(40) A room's breadth is 19 feet. If its length were increased by 3 feet, and its breadth decreased by 1 foot, its area would be unaltered. Find its length.

Ex. 20. **H.C.F.**

Find the H.C.F. of

(1) x^2y, xy^3. (2) a^2b^2, $2ab$. (3) $4a$, $2ab$.

(4) p^3q^4, p^4q^3. (5) $16x^3yz$, $12xyz^3$. (6) $9a^2$, $12abc$.

(7) $14a^3x$, $21a^2xy$. (8) $7a^3bc$, $8a^2b^3$. (9) $25a^2$, $40b^2c^2$.

(10) $16x^3y^2z$, x^2y, $4xy^3z^2$. (11) $14abc^2$, $7b^2c^3$, $21a^2b^2c^4$.

(12) pqr, $12p^2q^2r$, $15pr^2$. (13) $27ax^3y^2$, $36a^2x$, $45abxy^3$.

(14) $26ab^2$, $39a^3bc$, $52a^2b^2c^2$. (15) $40xy^2z$, $24x^2yz$, $15xyz^2$.

Ex. 21. **L.C.M.**

Find the L.C.M. of

(1) xyz, $3z^2$. (2) a^2b^3, abc. (3) $2x^3y$, $4xy^2$.

(4) $3xy$, $4x^2y^3$. (5) $9a^3b^3$, $3ab^2c$. (6) ax^3, $5abx^2$.

(7) xy, yz, zx. (8) l^2m, m^2n, n^2l. (9) a^2bc, ab^2c, abc^2.

(10) $3a^2$, $4b^2$, $6c^2$. (11) $21a^2b^2c^2$, $7ab$, $3ac$. (12) pqr, p^2, q^2r^2.

(13) $22ab^2$, $11a^3b$, $3a^2b^2$. (14) $6rst$, $15r^2t^7$, $5s^3$, $10rt^2$.

(15) $34x^2y^2$, $17xyz^5$, $3x^4$, z^3.

Ex. 22.

Find H.C.F. and L.C.M. of

(1) $6xyz$, $3x^2y^2$, $2x^2z$. (2) $2cab$, $4ac$, $3bc$.

(3) $13x^2yz$, $26xy^2z$, $2ax^2y^2$. (4) $8ac^2b$, $6a^2bc$, $4ab^2c$.

Find H.C.F. and L.C.M. of

(5) $17ax$, $34by$, $85cz$. (6) x^3, $3ax^2y$, $4ay^2$.

(7) $pqrs$, $12r^2s^2$, $9p^2q^2$. (8) lm^2, $2lmn$, $4ln^2$.

(9) $7ab^2cd$, $49a^2d$, $21abc^2$. (10) $11xyz$, $7ax^3$, $15axz^2$, $3xy^2$.

[Additional examples may be obtained by finding the L.C.M. of the expressions given under H.C.F. and *vice versa*.]

Ex. 23. FRACTIONS.

Reduce to lowest terms:

(1) $\frac{a^7b}{a^4b^3}$. (2) $\frac{2a^3bc}{3a^3b^2}$. (3) $\frac{4ax}{8x}$. (4) $\frac{3xyz}{12x^2y^2z^2}$.

(5) $\frac{2a^4b^3}{ab^2}$. (6) $\frac{32a^4b^3}{16xy}$. (7) $\frac{3ab}{7b^3}$. (8) $\frac{9abx^2}{27a^2b^2x^2}$.

(9) $\frac{26a^3bc^3x}{39a^2bcx^3}$. (10) $\frac{6yz}{15xyz}$. (11) $\frac{4}{2x}$. (12) $\frac{lmn^2}{l^2mn^2}$.

(13) $-\frac{3^2 . x^8}{27x^6y}$. (14) $\frac{(-2)^3 . x^6yz^2}{4^2 . xyz}$. (15) $\frac{64ab^2cx^8y^7z^6}{72za^2xb^3y^4c}$.

Ex. 24.

Simplify by performing the multiplications or divisions:

(1) $\frac{ab}{2x} \times \frac{x^2}{3a^2}$. (2) $\frac{3a^2b}{2y^3z} \times \frac{yz}{ab^2}$. (3) $\frac{x^3pt}{x^2q} \times \frac{qrt}{r^2p^2t^2}$.

(4) $\frac{2xy}{abc} \div \frac{2xy}{ab}$. (5) $\frac{2xy^2}{ab} \div \frac{2xy}{ab}$. (6) $\frac{3c^2zn}{abc} \div \frac{n^2z^2c^2}{a^3}$.

(7) $\frac{abx}{16c^2} \times \frac{2cy}{3a^3} \times \frac{24ca^2}{bxy}$. (8) $2 \times \frac{x^2}{y} \times \frac{y^2}{2a} \times \frac{a^2}{x} \div xyz$.

(9) $\frac{3z^7}{2x} \times \frac{x^4}{z^2} \div \frac{z^5x^2}{3}$. (10) $\frac{1}{6} \times \frac{24a^2}{39x} \times \frac{13bx}{8abc} \div \frac{5a}{3cx}$.

(11) $\frac{7ab^2c}{8x^2y^2z} \div \frac{-8abc}{7xyz}$. (12) $\frac{l^2m^3}{p^2q^2} \div \frac{-l^2m^2}{p^3q^4}$.

(13) $\frac{24ax}{7y} \div \frac{1}{2} \times \frac{49}{48} \cdot \frac{y}{a}$. (14) $\frac{6a^3bx}{5ayz^2} \times \frac{15by^2z}{2a^2x^4} \div \frac{9b^2y^2}{x^3z}$.

Ex. 25.

Oral. Express with common denominator:

(1) $\frac{x}{2}, \frac{x}{3}$. (2) $\frac{a}{4}, \frac{2a}{7}$. (3) $\frac{b}{2}, \frac{b}{4}$.

(4) $\frac{2y}{10}, \frac{4y}{15}$. (5) $\frac{x}{2}, \frac{2x}{3}, \frac{x}{4}$. (6) $\frac{a}{5}, \frac{a}{10}, \frac{2a}{15}$

(7) $\frac{5b}{6}, \frac{b}{3}, -\frac{b}{2}$. (8) $\frac{3}{x}, \frac{a}{2x}$. (9) $\frac{2}{y^2}, \frac{4}{xy}$.

(10) $\frac{a}{bc}, \frac{b}{ca}$. (11) $\frac{x}{yz}, \frac{y}{zx}, \frac{z}{xy}$. (12) $\frac{r}{p^2q}, \frac{s}{p}$.

(13) $\frac{3x}{y^2}, \frac{2y}{x^2}$. (14) $-x, \frac{5}{y}, \frac{3}{z}$. (15) $\frac{a^2}{b^3}, \frac{2a}{3b}, b^2$.

(16) $\frac{lk^2}{2}, \frac{3k}{4x}, \frac{1}{x^2}$. (17) $\frac{2c^2}{a^2bc}, \frac{3ab}{b^2ca}, \frac{5a^2}{abc^2}$. (18) $\frac{1}{8xyz^3}, \frac{2x}{y^3}, \frac{7}{3xyz}$.

Ex. 26.

Perform the following additions or subtractions:

(1) $\frac{x}{2}+\frac{x}{3}$. (2) $\frac{a}{4}-\frac{a}{7}$. (3) $\frac{b}{2}+\frac{b}{5}-\frac{b}{3}$.

(4) $\frac{2y}{3}-\frac{y}{4}$. (5) $\frac{x}{2}-\frac{2x}{3}+\frac{3x}{4}$. (6) $\frac{2a}{3}-\frac{b}{4}+\frac{a}{6}$.

(7) $\frac{2x+1}{3}+\frac{x+2}{5}$. (8) $\frac{x+2}{4}-\frac{x-1}{8}$.

(9) $\frac{2x+3}{3}+\frac{x-2}{4}-\frac{5x+3}{6}$. (10) $\frac{2}{x}-\frac{1}{3x}$. (11) $\frac{4a}{3b}-\frac{a}{b}$.

(12) $\frac{3x^2}{ab}+\frac{x^2}{4ab}$. (13) $\frac{2a}{xb}-\frac{3}{x}$. (14) $\frac{2xy}{x^2}-\frac{y}{x}$.

(15) $\frac{a}{b}+\frac{b}{a}-\frac{a^2+b^2}{ab}$. (16) $a-\frac{y^3}{a^2}$. (17) $\frac{n}{2}-\frac{a^3}{n^2}$.

(18) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$. (19) $\frac{1}{a}-\frac{7}{3a}+\frac{2}{5a}$. (20) $\frac{2bc}{ax}-\frac{abc}{a^2x^2}+1$.

Ex. 27. INVOLUTION.

Write down the square of

(1) a^2b. (2) xyz^2. (3) $2ab^2c$. (4) $3a^3$.

(5) $7a^5b$. (6) $12x^4z$. (7) $-4ab^2cz$. (8) $-3a^3x^7$.

(9) $-5ax^2$ (10) $10a^7xy$. (11) $\frac{1}{2}x$. (12) $-\frac{2}{3}x^2yz$.

(13) $-\frac{1}{5x^3}$. (14) $\frac{5xb^2}{2ay}$. (15) $\frac{6yz}{7a^2b^3}$. (16) $-\frac{4a^3bc}{9x^3z^2}$.

Write down the cube of

(17) ab^2. (18) $2xy$. (19) $3ax^3$. (20) $3a^2y^3z$.

(21) $-xz^4$. (22) $-4lm^2$. (23) $5x^7$. (24) $-3y^3z^2$.

(25) $\frac{x}{2}$. (26) $-\frac{xyz}{3}$. (27) $\frac{2a^2bc}{lm}$. (28) $-\frac{4ab^3}{5x^5}$.

Write down the expanded form of

(29) $(2ab^3)^4$. (30) $(3xy^2)^4$. (31) $(-a^7)^5$.

(32) $(a^6bc)^4$. (33) $(2ab^2)^5$. (34) $(-pq^2)^7$.

(35) $(-p^2q^3)^8$. (36) $\left(-\frac{2ab^3}{3xy^2}\right)^3$. (37) $\left(\frac{xz^2}{yw^3}\right)^5$.

(38) $(-3)^4 . xy^2$. (39) $x(y^2z^3)^5$. (40) $-(p^3q^2)^3$.

(41) $2xyz . (3xz)^4$. (42) $a^2 . (-b)^3 . (ab^4)^5$.

(43) $x^2 . (-y^2)^3 . x^3 \div y^7$.

Ex. 28. EVOLUTION.

Write down the square root of

(1) a^6. (2) a^4b^2. (3) $a^8x^4y^4$. (4) $x^2y^6z^4$.

(5) $4a^2x^6$. (6) $9b^2c^6$. (7) $25c^2z^{12}$. (8) $a^{24}b^{16}c^2$.

(9) $16x^6y^{20}$. (10) $81y^{40}$. (11) $\frac{x^8}{16}$. (12) $\frac{4y^2z^8}{9}$.

(13) $\frac{169a^8}{25z^2}$. (14) $\frac{144b^6p^2}{196c^8}$. (15) $\frac{49l^4m^2n^2}{121p^2q^2r^4}$.

Write down the cube root of

(16) a^6. (17) b^3x^9. (18) $8x^{12}$. (19) $64z^3w^{15}$.

(20) $-b^{12}$. (21) $-27c^{15}d^{42}$. (22) $-125x^3z^3$. (23) $\frac{8l^3}{27m^6}$.

(24) $-\frac{64p^6r^{15}}{125x^{81}}$. (25) $\frac{x^9}{343y^3z^6}$. (26) $-\frac{216y^{12}}{125z^9w^3}$.

Write down the value of

(27) $\sqrt[4]{c^4d^{12}}$. (28) $\sqrt[5]{x^{10}y^{15}}$. (29) $\sqrt[5]{32x^{25}}$.

(30) $\sqrt[3]{-64l^3z^6}$. (31) $\sqrt[5]{-x^{20}y^5}$. (32) $\sqrt[9]{x^{27}z^{63}}$.

(33) $\sqrt[4]{\frac{x^4z^8}{16}}$. (34) $\sqrt[6]{\frac{64}{a^{12}b^{18}}}$. (35) $\sqrt{-\frac{x^{18}}{y^{27}z^9}}$.

Ex. 29. SIMULTANEOUS EQUATIONS.

Solve the following pairs of equations:

(1) $x+y=7$, $x-y=3$. (2) $x+y=210$, $x-y=64$. (3) $y-x=6$, $x+y=38$.

(4) $x+y=33$, $y-x=7$. (5) $x-y=4$, $x+y=9$. (6) $2x-y=10$, $x-y=3$.

(7) $x+3y=12$, $-x+4y=-5$. (8) $2x-3y=4$, $2x+y=7$. (9) $x-y=21$, $2x+y=3$.

(10) $2x+3y=8$, $5x-y=3$. (11) $2x+3y=37$, $x-13y=4$. (12) $6x+8y=35$, $2y-4x=6$.

(13) $6x+3y=48$, $3x-10y=1$. (14) $x+y=3$, $4y-3x=17$. (15) $2x+5y=-3$, $7x-10y=17$.

(16) $12x+7y=59$, $8x+13y=81$. (17) $x-8y=4$, $2x-8y=0$. (18) $4x+7y=5$, $6x+5y=2$.

(19) $2x+y=-12$, $x-7y=9$. (20) $8x+12y=20$, $18y-5x=-16$. (21) $x+y=8022$, $y=x-5846$.

(22) $5x-6y=4$, $3x=4y$. (23) $9y=7x$, $x-y=2$. (24) $x+8y=18$, $3x=20y+65$.

(25) $x+y=0$, $6x+7y=7$. (26) $4(x-2)=3(y+2)$, $3(x+2)=7(y+1)$.

Solve the following pairs of equations :

(27) $3(x+1)=5(y-1)$, $5(x-1)=3(y+1)$.

(28) $8x+4y=2$, $y=3-x$.

(29) $4x-y=0$, $5y=168x-37$.

(30) $10x=4-3y$, $3y=5x+1$.

(31) $5x-6y=17$, $y=x-10$.

(32) $5x+9y=3$, $9x+7y=10$.

(33) $x-y=2x-3y=6$.

(34) $x+y=25-3x=10$.

(35) $2x+3y=2=10x-9y$.

(36) $13x-5y=6$, $22x+17y=41$.

(37) $8y-3x=30$, $x=4y$.

(38) $1216x-374y=842$, $43x+78y=121$.

(39) $x+7=y+9=3x-4y$.

(40) $10x+4y=3$, $20y-5x=4$.

Each of the equations (41), (42), (43), (44), (45) is to be taken with each of the equations (*a*), (*b*), (*c*), (*d*), (*e*), and the pair of simultaneous equations so obtained is to be solved. (These equations are especially suitable for graphical solution.)

(41) $x-y=0$.
(42) $3y-x=1$.
(43) $x-2y=2$.
(44) $3x-y=9$.
(45) $x+1=0$.

(*a*) $x+y=1$.
(*b*) $x+3y+1=0$.
(*c*) $x+2y=4$.
(*d*) $2y=3$.
(*e*) $x=3$.

Ex. 30.

Solve the following pairs of equations :

(1) $\frac{1}{2}x+\frac{1}{3}y=3$, $x+y=7$.

(2) $\frac{x}{3}-\frac{y}{2}=1$, $\frac{y}{11}=11-\frac{x}{4}$.

(3) $\frac{x+y}{2}=9$, $\frac{x-y}{3}=4$.

(4) $\frac{1}{2}x+3y=2$, $x+4y=0$.

(5) $\left.\begin{aligned}\tfrac{1}{2}(x+y)&=4\tfrac{1}{2},\\ \frac{x}{4}-y&=1.\end{aligned}\right\}$

(6) $\left.\begin{aligned}x+y&=3,\\ \frac{x}{7}+y&=0.\end{aligned}\right\}$

(7) $\left.\begin{aligned}\cdot3x+\cdot5y&=\cdot23,\\ 6x+5y&=2\cdot6.\end{aligned}\right\}$

(8) $\left.\begin{aligned}\cdot1x+3y&=2\cdot6,\\ x-1\cdot6y&=10\cdot2.\end{aligned}\right\}$

(9) $\left.\begin{aligned}3x+2y&=9\cdot29,\\ x-2y&=\cdot43.\end{aligned}\right\}$

(10) $\left.\begin{aligned}6x+2y&=6\cdot8,\\ y&=2x-\cdot1.\end{aligned}\right\}$

(11) $\left.\begin{aligned}x+y&=5\cdot83,\\ 7x+3y&=35\cdot09.\end{aligned}\right\}$

(12) $\left.\begin{aligned}\cdot5x+\cdot3y&=1\cdot29,\\ x+\cdot7y&=3.\end{aligned}\right\}$

(13) $\left.\begin{aligned}x&=\frac{2}{3}y,\\ 9y-11x&=80.\end{aligned}\right\}$

(14) $\left.\begin{aligned}y&=\frac{5}{2}x,\\ y-x&=x+7.\end{aligned}\right\}$

(15) $\left.\begin{aligned}\frac{x}{3}-\frac{2x-y}{4}&=-2,\\ \frac{x-3y}{6}&=5-2(5y-x).\end{aligned}\right\}$

(16) $\dfrac{3x+5}{4}=\dfrac{2y-7}{5}=0.$

(17) $\dfrac{x-y}{3}=\dfrac{x}{4}=y-6.$

(18) $\dfrac{x-y}{2}=\dfrac{2x}{5}=6-y.$

(19) $\left.\begin{aligned}\cdot3(7x+2y)-\cdot08&=\cdot7(x+5),\\ y-\cdot2x&=3(y-1).\end{aligned}\right\}$

(20) $\begin{aligned}\cdot5x&=\cdot4(x+y)-\cdot3,\\ \cdot2y&=\cdot7(y-x)+\cdot1.\end{aligned}$

(21) $2x+y=6-x=0.$

(22) $x=\dfrac{y-1}{3}=\dfrac{x-9}{2}.$

(23) $\dfrac{2x+1}{2}=\dfrac{x-y}{3}=\dfrac{y}{2}.$

(24) $\dfrac{x-8y}{4}=\dfrac{x+2}{3}-8=160y-(x-10).$

Ex. 31. GRAPHS.

(1) Make tables to show the values of y corresponding to $x=0, 1, 2, 3, 4, 5, 6$ in the following cases:

(i) $y=2x+1$, (ii) $y=3x+2$, (iii) $y=2x-3$, (iv) $y=\frac{1}{2}x+1$, and draw the graphs on squared paper.

(2) Draw with the same origin and axes the graphs corresponding to

$$\text{(i) } y=\frac{1}{2}x, \quad \text{(ii) } y=\frac{1}{2}x+2, \quad \text{(iii) } y=\frac{1}{2}x-1.$$

(3) Draw with the same origin and axes the graphs corresponding to

$$\text{(i) } y=3x, \quad \text{(ii) } y=3x-4, \quad \text{(iii) } y=3x+1.$$

(4) In Fig. 1 on opposite page the graph shows the number (y) of inches corresponding to a number (x) of centimetres. Use the graph to find out:

(i) How many inches in ·4 centimetres? in 4 centimetres?

(ii) How many inches in ·75 centimetres? in 7·5 centimetres?

(iii) How many centimetres in ·1 inch? in 1 inch? in 3·5 inches?

(5) Construct a table showing the values of x^2 for

$$x=-10, \ -9, \ \ldots 0, \ 1, \ \ldots 10.$$

Draw the corresponding graph from

$$x=-4 \text{ to } x=+4.$$

(6) Show in a table the values corresponding to

$$x=0, \ 1, \ 2, \ \ldots 6$$

of the three functions

$$x^2+x; \quad x-\frac{1}{2}x^2; \quad 2x^3-x^2+x-1.$$

(7) Use Fig. 2 on opposite page to find what x^2 is if x is ·35, and to find what x is if x^2 is ·9. Draw the graph of $y=x^2$ between $x=0$ and $x=1$ on as large a scale as possible and use to find

$$(\cdot 75)^2, \quad (7\cdot 5)^2, \quad (\cdot 63)^2, \quad \sqrt{\cdot 5}, \quad \sqrt{50}, \quad \sqrt{\cdot 64}.$$

Check by using the tables or otherwise.

(8) Draw the graphs for $x=0, \ \cdot 1, \ \cdot 2, \ \cdot 3, \ \ldots \cdot 9, \ 1,$ of the functions (i) x^3, (ii) $1+2x+x^2$.

(9) Draw with the same origin and axes the graphs of

$$\text{(i) } y+x=1; \quad \text{(ii) } y+x=0; \quad \text{(iii) } y+x=-1.$$

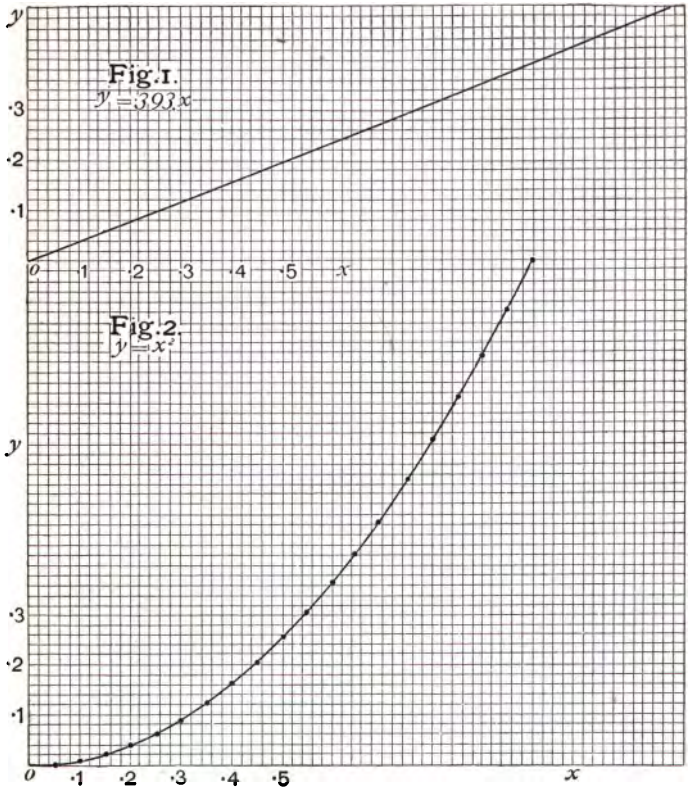
(10) Draw, by joining the points where the lines cut the axes, the graphs of:

(i) $y=2x-1$; (ii) $2x+y=6$; (iii) $2x-y=4$:

(iv) $x = 3y + 3$; (v) $x + y = 3$; (vi) $x - y = 1$;

(vii) $\frac{x}{3} + \frac{y}{2} = 1$; (viii) $\frac{x}{4} - \frac{y}{3} = 1$; (ix) $y = \frac{1}{3}(x + 3)$;

(x) $y = 2x - \frac{1}{2}$; (xi) $y = \cdot 2x + \cdot 3$; (xii) $y = \cdot 4x - \cdot 5$.



(11) Draw the graph of

$$y = \frac{22}{15}x \text{ (i.e. } y = 1 \cdot 47x),$$

which gives the connection between the number (x) of miles per

hour and the number (y) of feet per second in the same speed, and use it to find:

(i) How many feet per second in 10 miles an hour? in 4 miles an hour?

(ii) How many miles per hour in 30 ft. per second (100 yds. in 10 seconds)?

(12) Draw the graph

$$y = 32 + \frac{9}{5}x,$$

which gives the connection between the number (y) of degrees fahrenheit and the number (x) of degrees centigrade in the same temperature. [Draw the graph putting $x = -5$, $y = 23$ at the bottom of the paper.]

Use the graph to find:

(i) How many degrees fahrenheit in 20 degrees centigrade?

(ii) How many degrees centigrade in 55 degrees fahrenheit?

Ex. 32. SOLUTION OF EQUATIONS BY MEANS OF GRAPHS.

(1) Draw with the same axes the graphs of

$$y - 2x = 1, \quad x + y = 2, \quad x - y = 3,$$

and deduce the solutions of the three pairs of equations:

$$\left.\begin{matrix} y - 2x = 1, \\ x + y = 2. \end{matrix}\right\} \qquad \left.\begin{matrix} x + y = 2, \\ x - y = 3. \end{matrix}\right\} \qquad \left.\begin{matrix} x - y = 3, \\ y - 2x = 1. \end{matrix}\right\}$$

(2) Draw with the same axes the graphs of

$$y - 3x = 0, \quad y - x = 2, \quad 3x + 2y = -6,$$

and deduce the solutions of the three pairs of equations:

$$\left.\begin{matrix} y - 3x = 0, \\ y - x = 2. \end{matrix}\right\} \qquad \left.\begin{matrix} y - x = 2, \\ 3x + 2y = -6. \end{matrix}\right\} \qquad \left.\begin{matrix} 3x + 2y = -6, \\ y - 3x = 0. \end{matrix}\right\}$$

(3) Draw with the same axes the graphs of

$$y=\frac{1}{2}x, \quad y=4-\frac{1}{3}x, \quad y+2=\frac{1}{4}x,$$

and deduce the solution of three pairs of equations.

(4) Draw with the same axes the graphs of

$$3y=2x+3, \quad y-x=2, \quad 2x=5,$$

and deduce the solution of three pairs of equations.

(5) Draw the graph of $y=x^2$ (from $x=-2$ to $x=+2$), and with the same axes the graph of $y=x+1$.

Hence solve the equations:

$$\left.\begin{aligned} x^2-y&=0,\\ x-y+1&=0. \end{aligned}\right\}$$

(6) Use the graph drawn in Ex. 5 to solve the equations:

$$\left.\begin{aligned} 2x&=3y,\\ y&=x^2. \end{aligned}\right\}$$

(Further examples of this method will be found in Miscellaneous Graphs.)

Ex. 33. GRAPHS OF STATISTICS.

Exhibit the following statistics graphically:

(1) Cost of life-insurance at different ages.

Age in years, - -	25	30	35	40	45	50	55	60
Premium in £, - -	2·3	2·6	2·9	3·3	3·8	4·5	5·7	7·2

Estimate the premium at ages 32 and 47.

(2) Fall of barometer due to increase of height above sea-level.

Height in thousands of feet,	0	5	10	15	20	25	30
Height of barometer in inches,	30	24·9	20·6	17·1	14·2	11	9·8

Estimate the heights of the barometer at 7000 feet and 1900 feet above sea-level.

(3) Comparison of the populations of England and Wales, Scotland, and Ireland, at various dates (population in millions).

Years, - -	1821	1831	1841	1851	1861	1871	1881	1891	1901
England and Wales,	12·0	13·9	15·9	17·9	20·1	22·7	26·0	29·0	32·5
Scotland, -	2·1	2·4	2·6	2·9	3·1	3·4	3·7	4·0	4·47
Ireland, - -	6·8	7·8	8·2	6·6	5·8	5·4	5·2	4·7	4·45

Estimate when the population of England and Wales was just 25 millions, and when the populations of Scotland and Ireland were equal.

(4) Number of pages in successive editions of the *Encyclopædia Britannica.*

Edition, -	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
Thousands of pages,	2·7	8·6	14·6	16	16	16	17	18	21·6	31

(5) The ageing of electric glow lamps.

Hundreds of hours run,	1	2	3	4	5	6	7	8
Candle-power, - -	16·0	15·9	15·8	15·65	15·4	15·2	15·0	14·7

(6) Number of successful candidates for honours in the Mathematical Tripos at Cambridge.

Year, -	1800	1810	1820	1830	1840	1850	1860	1870	1880	1890	1900
Number,	31	45	52	98	147	122	121	115	99	105	82

(7) Number of successful candidates for honours in the Natural Sciences Tripos at Cambridge.

Year, -	1882	1885	1888	1891	1894	1897
Number, -	54	45	81	89	95	92

(8) The world's production of petroleum in various years (measured in millions of 42-gallon barrels).

Year,	1875	1876	1877	1878	1879	1880	1881	1882	1883	1884	1885
Millions of barrels,	8·9	9·1	13·0	15·0	20·1	24·8	29·7	35·8	34·4	23·6	21·6

(Further examples will be found in Miscellaneous Graphs.)

Ex. 34. SIMULTANEOUS EQUATIONS.

(*More than two unknowns.*)

Solve the equations:

(1) $\left.\begin{aligned} x+y+z&=7,\\ 3x+y-z&=3,\\ 2x+4y+z&=12. \end{aligned}\right\}$

(2) $\left.\begin{aligned} 2x-y+z&=3,\\ x+2y+z&=12,\\ 4x-3y+z&=1. \end{aligned}\right\}$

(3) $\left.\begin{aligned} x+y+z&=13,\\ 3x+y-3z&=5,\\ x-2y+4z&=10. \end{aligned}\right\}$

(4) $\left.\begin{aligned} x-3y+z&=10,\\ 2x-7y-5z&=-2,\\ x+y-2z&=5. \end{aligned}\right\}$

(5) $\left.\begin{aligned} 4x+y+z&=2,\\ x+2y&=1,\\ z-x-5y&=5. \end{aligned}\right\}$

(6) $\left.\begin{aligned} x+y&=2,\\ y+z&=-2,\\ z+x&=12. \end{aligned}\right\}$

(7) $\left.\begin{aligned} x+y&=5,\\ y+z&=3,\\ z+x&=7. \end{aligned}\right\}$

(8) $\left.\begin{aligned} x-y&=3,\\ y-z&=5,\\ z+x&=9. \end{aligned}\right\}$

(9) $\left.\begin{aligned} x+y&=3,\\ y+z&=3,\\ z+w&=7,\\ -w+x&=4. \end{aligned}\right\}$

(10) $\left.\begin{aligned} x+y+z&=6,\\ 2y+z&=9,\\ z-2y&=1. \end{aligned}\right\}$

(11) $\left.\begin{aligned} x+y+z+w&=12,\\ x-y+z&=9,\\ 2x+y&=8,\\ y-4z&=5. \end{aligned}\right\}$

(12) $\left.\begin{aligned} 6x+y&=13,\\ y+6z&=19,\\ z+x-y&=4. \end{aligned}\right\}$

(13) $\left.\begin{aligned} 4x-3w&=10,\\ w+3v&=19,\\ 3x+y&=13,\\ 2y-3u&=11,\\ 2y-5u&=5. \end{aligned}\right\}$

(14) $\left.\begin{aligned} 10x+8y-9z&=10,\\ 12x+2y+11z&=13,\\ 2x+10y-25z&=0. \end{aligned}\right\}$

(15) $\left.\begin{aligned} 3+x&=5-4w,\\ z+x&=3w,\\ 7w&=z+2. \end{aligned}\right\}$

(16) $\left.\begin{aligned} 3(z-1)&=2(y-1),\\ 4(y+x)&=9z-4,\\ 2y&=7(5x-3z)+9. \end{aligned}\right\}$

(17) $\left.\begin{aligned} \frac{x+1}{2}&=\frac{1}{3}+\frac{2y}{9},\\ x+\frac{y}{2}+4z&=\frac{1}{4}(y+2z+1),\\ x+2z&=1. \end{aligned}\right\}$

(18) $\left.\begin{aligned} x+2y&=2x+y-27\\ &=z-3x\\ &=6. \end{aligned}\right\}$

(19) $\left.\begin{aligned} 2x-\frac{3y}{2}+z&=-\frac{1}{3},\\ x-3z+2-z&=3y-4z,\\ \frac{x}{3}-\frac{z}{3}+2y-z&=3y-\frac{4z}{3}+4-y. \end{aligned}\right\}$

(20) $\left.\begin{aligned} \frac{10x+2y-5z}{5}&=\frac{4x+3y-3z}{9},\\ 10x+2y-5z&=4x+3y-3z-8,\\ \frac{10x+2y-5z}{10}+\frac{4x+3y-3z}{3}&=\frac{x+y+z}{4}. \end{aligned}\right\}$

(21) $\left.\begin{aligned} 7x-11y+2z&=10,\\ -10x+3y+5z&=-15,\\ 12x-y-6z&=31. \end{aligned}\right\}$

(22) $\left.\begin{aligned} \frac{x}{6}+\frac{y}{3}+\frac{z}{4}&=36,\\ \frac{x}{9}+\frac{y}{15}+\frac{z}{20}&=10,\\ \frac{x}{4}+\frac{y}{2}+\frac{z}{10}&=43. \end{aligned}\right\}$

Ex. 35. PROBLEMS.

(1) The sum of two numbers is 1384 and their difference is 62: find them.

(2) Find two numbers whose sum is 635 and their difference 321.

(3) Three times the sum of two numbers is 372 and twice their difference is 48: find them.

(4) If 8 be added to the larger of two numbers the result is three times the other; the difference of the numbers is 10: find them.

(5) Two numbers are such that five times the smaller is equal to three times the larger; the sum of the numbers is 888: find them.

(6) If the sum of two numbers be added to three times their difference the result is 18; if twice the sum be added to their difference the result is 26: find them.

(7) Half the sum of two numbers is 51; their difference is 6: find them.

(8) Half the difference of two numbers is 2; their sum is 7: find them.

(9) One third of the sum of two numbers is 24, while a quarter of their difference is 12: find the numbers.

(10) The sum of two numbers is 10 more than their difference; if 3 be added to the larger the result is 6 times the smaller: find the numbers.

(11) £832 is divided between A and B so that B gets £68 more than A: how much does each get?

(12) £200 is divided between A and B so that A has £3 more than B: how much has each?

(13) Divide £375 between A and B, so that A gets £35 less than twice what B gets.

(14) A, B, C, D have £480 between them: A has twice as much as B, and C has 7 times as much as D; if C has £50 more than A, how much has each?

(15) B, C, D have £700 between them. D has £60 more than B and C have between them: how much has each if C is known to have £80 more than B?

(16) A man buys 5 horses and 3 cows for £167: he could have bought 3 horses and 5 cows for £145: what is the cost of each?

(17) £200 will buy 5 cows and 20 sheep; the same amount will buy 8 cows and 8 sheep: what is the cost of each?

(18) A party of 33 cyclists were mounted, some on tandem and the rest on single machines. If there were 12 more single machines than tandems, how many were there of each?

(19) Five years hence a father will be 3 times as old as his son; their ages added make 66 years: how old are they?

(20) A bicycle's age is one-eighth of that of its owner. It is not to be sold for 5 years, when its age will be one-third of his: how old is the owner now?

(21) A is 4 years older than B; six years ago the sum of their ages was two-thirds what it is now: find their ages.

(22) A is twice as old as B; 16 years ago he was 4 times as old as B: find their ages.

(23) If 2 be taken from the numerator and also from the denominator of a fraction the result is equal to $\frac{1}{2}$; if 1 be taken from the numerator only the result is the same: find the fraction.

(24) To the numerator and denominator of a fraction 1 is added; the result is $\frac{1}{3}$: if 9 be taken from the denominator the result is $\frac{1}{2}$: find the fraction.

(25) A book about a certain war has 2 pages for every 3 days that the war lasted. The average number of paragraphs on each page is $1\frac{2}{3}$, and there are 107 more paragraphs in the book than days that the war lasted. What was the number of days?

(26) The sum of three numbers is 360. The difference between the first and second is equal to the difference between the second and third; while the difference between the first and third is 40 more than the second number: find the numbers.

(27) The sum of three numbers is 360. The difference between the first and second is equal to the sum of the second and third; the sum of the first and third is equal to three times the second: find the numbers.

(28) Find three numbers such that the three sums obtained by adding them two at a time are 46, 57, 65.

(29) Find four numbers such that the four sums obtained by adding them three at a time are, 9, 10, 11, and 12.

(30) Divide 126 into 4 parts, such that if 2 be added to the first, 2 subtracted from the second, the third multiplied by 2, and the fourth divided by 2, the results may all be equal.

Ex. 36. MULTIPLICATION.

Find the product of

(1) $x+2,\ x+5.$ (2) $a-1,\ a+3.$ (3) $y+6,\ y-3.$

(4) $a-2,\ a+2.$ (5) $b+7,\ b-4.$ (6) $-a+5,\ a-8.$

(7) $a+3,\ b-1.$ (8) $x-2,\ a+4.$ (9) $x+4,\ -x+3.$

(10) $a+b,\ a-b.$ (11) $x-y,\ x+2y.$ (12) $x-c,\ 2x+2c.$

(13) $xy-5,\ xy+3.$ (14) $ab-6x,\ ab+6x.$ (15) $2x-5a,\ x+a.$

Find the value of

(16) $(x+y)(2x-y).$ (17) $(a+4b)(2x-b).$

(18) $(x^2+y^2)(x^2-y^2).$ (19) $(2ax+b)(ax-b).$

(20) $(3-ab)(4+ab).$ (21) $(3axy+2x^2)(axy-x^2).$

[A number of examples similar to the above and intended for oral work are given in Exercise 41.

In some of the following the checks suggested should be used, and especially in numbers 32 to 37.]

Multiply together

(22) $3x^2+x+1,\ x+2$; check $x=1.$

(23) $x^3+x^2+x+1,\ x-1$; check $x=2.$

(24) $3a^3+2a^2-3a+1,\ 2a-1$; check $a=1.$

(25) $4b^2+b-1,\ b-2$; check $b=1.$

(26) $7x^6-5x^5+3x^4-x^3-2x^2-1,\ 3x^2-1$; check $x=1.$

(27) $a^2+ab+b^2,\ a^2-ab+b^2$; check $a=b=1.$

(28) $2a^2+3a^3-2,\ a+a^2+3$; check $a=1.$

(29) $y^3-y^2+y-1,\ y+1$; check $y=2.$

Multiply together

(30) x^2+xy+y^2, x^2-y^2; check $x=2$, $y=1$.

(31) $1+x+x^2+x^3$, $3x^2+2x-1$; check $x=1$.

(32) x^3+2x^2+2x+1, x^2+3x+1; check by putting $x=10$, and comparing with the arithmetical process of multiplication.

(33) $2x^3+x^2+2x+2$, x^2+x+3; check $x=10$.

(34) y^3+3y^2+2, $2y^2+2y+1$; check $y=10$.

(35) x^4+3x^2+1, x^2+3x+1; check $x=10$.

(36) $3x^2+6x+4$, $2x^2+x+8$; check $x=10$.

(37) x^3+8x^2+1, x^3+2x^2+5x+4; check $x=10$.

Multiply

(38) $a^5-5a^3+2a^2-1$ by $a^5+5a^3-2a^2+1$; check $a=1$.

(39) $a^2+b^2+c^2+bc+ca-ab$ by $a+b-c$; check $a=2$, $b=1$, $c=1$.

(40) $4x^2+9y^2+z^2-2xz-3yz-6xy$ by $2x+3y+z$; check $x=y=z=1$.

Find the value of

(41) $(x+y+z)^2$. (42) $(x-y)^2(x+y)$. (43) $(a+b)^3$.

(44) $(2a-b)(a-2b)(a+b)$. (45) $(x+2y+3z)^2$.

[Many of the above examples are suitable for the method of detached coefficients. This method should be employed in the following examples.]

(46) Calculate the powers of $a+b$ up to $(a+b)^{10}$, and state the rule by which the coefficients of one power can be obtained from those of the previous one.

(47) Calculate the powers of $a-b$ up to $(a-b)^5$. How can they be obtained from the powers of $a+b$?

(48) Calculate $(a+b)^6(a-b)^4$.

(49) Find the value of $(x^2+x+1)^4$.

(50) Find the value of $(x^3+x^2+x+1)^3$.

Ex. 37. APPLICATIONS TO GEOMETRY.

If any straight line AB be divided at C, show that

(1) $AB^2 = AB \cdot AC + AB \cdot BC$.
(2) $AB \cdot AC = AC^2 + BC \cdot AC$.
(3) $AB^2 = AC^2 + 2AC \cdot BC + BC^2$.
(4) $AB^2 + AC^2 = 2AB \cdot AC + BC^2$.

If C is the middle point of AB and P any point in BC, prove that

(5) $AP \cdot PB = AC^2 - CP^2$.
(6) $AP^2 + PB^2 = 2AC^2 + 2CP^2$.
(7) $AP^2 = 4BC^2 - 4BC \cdot BP + PB^2$.

If AB is bisected at C, and P any point on AB produced, prove that

(8) $AP \cdot PB = CP^2 - CA$.
(9) $AP^2 + PB^2 = 2AC^2 + 2CP^2$.

(10) If A, B, C, D are four points in order on a straight line, prove that

$$AC \cdot BD = AB \ CD + BC \cdot AD.$$

(11) If AB is 2 feet long and is divided in the ratio 2 : 3 internally at X and externally at Y, find the length of XY.

If AB is divided internally at X, and externally at Y in the ratio $l : m$,

(12) Find the length of XY.
(13) Prove that $AB(AX + AY) = 2AX \cdot AY$.
(14) Prove that $XY(AY + BY) = 2AY \cdot BY$.
(15) If M is the mid point of AB, prove that $MX \cdot MY = MA^2$.

Ex. 38. DIVISION.

Divide

(1) $12x^7 - 9x^5 + 18x^3$ by $3x^2$.
(2) $4x^6y^2 + 12x^5y^3 + 14x^4y^4$ by $2x^4y^2$.
(3) $x^7y - 3x^6y^2 + 4x^5y^3$ by x^4y.

Divide

(4) $7x^4 - 33x^3y + 14x^2y^2$ by x^2.

(5) $a^3b^2cd + ab^3c^2d + abc^3d^2 + a^2bcd^3$ by $abcd$.

(6) $x^2 + 4x + 4$ by $x + 2$; check $x = 1$.

(7) $x^2 - 5x + 6$ by $x - 3$; check $x = 1$.

(8) $x^2 + 5x + 6$ by $x + 3$; check $x = 10$.

(9) $x^2 + 6x + 9$ by $x + 4$; check $x = 10$.

(10) $3x^2 + 5x + 3$ by $3x + 2$; check $x = 10$.

(11) $x^2 + 7x + 12$ by $x + 3$; check $x = 10$.

(12) $2x^2 - 8x - 5$ by $2x + 1$; check $x = 10$.

(13) $x^2 - 25$ by $x - 5$; check $x = 1$.

(14) $4x^2 - 81$ by $2x - 9$; check $x = 1$.

Divide each of the expressions (15), (16), (17), (18) by each of the expressions (α), (β), (γ), (δ):

(15) $x^4 - 5x^3 + 5x^2 + 5x - 6$,
(16) $x^4 - 2x^3 - 7x^2 + 8x + 12$,
(17) $x^4 - 1$,
(18) $x^6 - 14x^4 + 49x^2 - 36$.

(α) $x + 1$,
(β) $x - 1$,
(γ) $x - 2$,
(δ) $x - 3$.

(19) Divide by $a - b$ each of the following:

$$a^3 - b^3,\ a^4 - b^4,\ a^5 - b^5,\ a^7 - b^7,\ a^n - b^n.$$

(20) Divide by $a + b$ each of the following:

$$a^3 + b^3,\ a^4 + b^4,\ a^5 + b^5,\ a^6 + b^6,\ a^{2n+1} + b^{2n+1}.$$

Ex. 39.

Divide

(1) $x^3 + 4x^2 + x - 6$ by $x^2 + 5x + 6$.

(2) $2x^3 - x^2 - 12x - 6$ by $2x^2 - 5x - 3$.

(3) $x^4 - 2x^3 - 7x^2 + 8x + 12$ by $x^2 - 2x - 3$.

Divide each of the expressions (4), (5), (6), (7) by each of the expressions (α), (β), (γ), (δ):

(4) $3x^3 + 2x^2 - 9x - 28$,
(5) $6x^4 + 7x^3 - 64x^2 + 23x + 28$,
(6) $6x^3 + 13x^2 - 51x - 28$,
(7) $6x^5 + 13x^4 - 57x^3 - 41x^2 + 51x + 28$.

(α) $x^2 + 3x - 4$,
(β) $2x^2 + 9x + 4$,
(γ) $6x^2 - 11x - 7$,
(δ) $3x^2 + 2x - 8$.

Divide

(8) $x^3 - y^3 + z^3 + 3xyz$ by $x - y + z$.

(9) $a^3 + b^3 + c^3 - 3abc$ by $a + b + c$.

(10) $x^4 + x^2y^2 + y^4$ by $x^2 + xy + y^2$.

(11) $27x^3 - 8y^3$ by $9x^2 - 6xy + 4y^2$.

(12) $x^6 + x^5a - 12x^4a^2 + 19x^3a^3 + 15x^2a^4 - 14xa^5 + 2a^6$
by $x^2 + 4xa - 2a^2$.

(13) $a^5 - a^4b - 4a^3b^2 - a^3 + 4a^2b^3 + 3a^2b - a^2 - 2ab^2 - 2ab + 1$
by $a^2 + 2ab - 1$.

(14) $1 + 2x + 3x^2 + 4x^3 + 3x^4 + 2x^5 + x^6$ by $1 + x + x^2 + x^3$.

(15) Find quotient and remainder when $x^2 + px + q$ is divided by $x - a$.

(16) Find quotient and remainder when $x^2 + a_1x + a_2$ is divided by $x - t$.

(17) Divide $x^5 + a_1x^4 + a_2x^3 + a_3x^2 + a_4x + a_5$ by $x - y$.

In examples (18) to (22) the expressions are first to be arranged in powers of x, and, secondly, in powers of y, and the quotients and remainders found in the two cases.

(18) $(x^2 + xy + y^2) \div (x + y)$. (19) $(x^2 + 2xy + y^2) \div (x + y)$.

(20) $(x^5 + 2x^4y + 3x^3y^2 + 4x^2y^3 - xy^4 + y^5) \div (x^2 - 3xy + y^2)$.

(21) $(x^8 - y^8) \div (x^2 - xy + y^2)$.

(22) $\{x^5 - 7x^4y + 4x^3y^2 - 9x^2y^3 + xy^4 - y^5\} \div (x^3 - 3x^2y + xy^2 - y^3)$.

[Most of the above examples are suitable for the method of detached coefficients.]

Ex. 40. MISCELLANEOUS.

(1) Multiply $x + 5$ by $x^3 + 2x^2 - x + 1$.

(2) Divide $x^2 - 8x + 7$ by $x - 1$.

(3) Give the expanded form of

(i) $(ab^2)^8$; (ii) $(-2ab^3)^5$; (iii) $\sqrt{49x^4y^{12}}$.

(4) Solve the equations:

(i) $\left.\begin{aligned} 2x - y &= 3, \\ x + 3y &= 26; \end{aligned}\right\}$ (ii) $\left.\begin{aligned} \frac{x}{2} + \frac{y}{3} &= 10, \\ \frac{x}{7} - \frac{y}{9} &= 1. \end{aligned}\right\}$

(5) If £36 will buy either 5 cows and 3 sheep, or 3 cows and 9 sheep, find the cost of each.

---

(6) Add $3x - 1$ to the difference between $x^4 + 2x^3 + x^2 + 4x + 1$ and $x^3 + x^2 + 3x + 1$.

(7) Multiply $a + b + c$ by $a^2 + b^2 + c^2 - bc - ca - ab$.

(8) Find the H.C.F. and L.C.M. of

$$27a^4b^3c, \quad 36a^2bc^3, \quad 45ab^2c^2.$$

(9) State in symbols 'x is the odd number next above twice n.'

Write down three consecutive numbers of which y is the middle one, and find their sum.

(10) Find three numbers such that the three results of adding them in pairs are 29, 31, and 36.

---

(11) Subtract $1 - x^2$ from the excess of 2 over $x - 1$.

(12) Simplify $(x + y - z)(x - y + z) - (x + y + z)(x - y - z)$.

(13) Find in its simplest form the sum of the squares of $x - 1$ and $x + 1$.

(14) Multiply out $(x^2 + 2x + 3)(x^5 - 3x^4 + 2x^3 - 4x^2 + x - 2)$.

(15) (i) The sum of a number and 115 is equal to six times the number; find it.

(ii) Find two numbers which differ by 104 such that one is three times the other.

---

(16) Expand (i) $(-pq)^7$; (ii) $(-p^2q)^8 \times p \div (pq)^8$.

(17) Divide $x^3 - 3x^2y + 3xy^2 - y^3$ by $x + y$:

(i) arranging in powers of x;

(ii) arranging in powers of y.

(18) Simplify
$$(a+b+c)^3 - 3\{a(b-c)^2 + b(c-a)^2 + c(a-b)^2\}.$$

(19) Draw the graph of $y = 2x - 7$.
For what range of values of x is y positive?

(20) Solve graphically $\left.\begin{aligned} y + x &= 1, \\ 2(x-y) &= 1. \end{aligned}\right\}$

---

(21) If $x = \dfrac{a-b}{c-d}$, prove that $cx + b = dx + a$

(22) The two equations
$$\left.\begin{aligned} px + q &= rx + p \\ qx + p &= px + r \end{aligned}\right\}$$ give the same value of x;
show that unless $q = r$ it is necessary that $p = \dfrac{q+r}{2}$.

(23) Tabulate the values of $20x - 16x^2$ for values of x differing by ·2 from $x = 0$ until the expression becomes negative.

Draw the graph and estimate for what value of x other than $x = 0$ the expression becomes 0.

(24) What values have p and q, if $5x + 7 = p(x - q)$ for all values of x?

(25) Work out the following products:
(i) $(x+y)(x-y)(x^2+y^2)(x^4+y^4)$;
(ii) $(x-y)(y-z)(z-x)$;
(iii) $(x-y)(x^3+y^3)(x^2+xy+y^2)$.

---

(26) If n is a whole number, prove that $2n+1$ is an odd number. Write down three consecutive odd numbers of which it is the greatest, and find their sum.

(27) If A's age is a years and B's age b years, what was the sum of their ages x years ago?

(28) A fish weighs 9 lbs. and half its own weight: how heavy is it?

(29) Prove that $(x^2+y^2)(z^2+w^2) = (xz-yw)^2 + (xw+yz)^2$
$= (xz+yw)^2 + (xw-yz)^2$;
and test by finding the value of each expression when $x = 1$, $y = -1$, $z = 2$, $w = 3$.

(30) Find the H.C.F. and L.C.M. of

(i) a^3b, ab^2, a^4b^4;
(ii) $9x^2y$, $27xy^3$, 36;
(iii) $39z^2w$, $26zw^2$, $91z^2w^2$.

If $s = ut - 16t^2$ in Nos. (31), (32), (33):

(31) Find u if $s = 36$, $t = 2$.

(32) Find u if $s = -1160$, $t = 10$.

(33) Find u if $s = 80$, $t = 3$.

If $v^2 - u^2 = 2fs$ in Nos. (34), (35), (36):

(34) Find s if $v = 20$, $u = 5$, $f = 3$.

(35) Find f if $v = 20$, $u = 40$, $s = 100$.

(36) Find s if $v = 60$, $u = 10$, $f = 3$.

(37) If $w = 42ab^2$, find a when $w = 420$ and $b = 2$.

(38) Find the radius r of a circle whose circumference c is 22 ft. from the formula $c = \frac{44}{7} \cdot r$.

(39) Find two numbers which differ by 8, such that half their sum is the same as twice their difference.

(40) A boy's age 5 years hence will be four times what it was 10 years ago. Find his present age.

Ex. 41. PRODUCTS OF BINOMIAL FACTORS.

Oral.

Each expression (1), (2), (3), (4), (5) is to be multiplied separately by each expression (*a*), (*b*), (*c*), (*d*), (*e*) in the same group; similarly for the other groups.

(1)	$x+1$,	(*a*)	$x+3$,	(6)	$x-8$,	(*a*)	$x-2$,
(2)	$x+5$,	(*b*)	$x+4$,	(7)	$x-1$,	(*b*)	$x-6$,
(3)	$x+2$,	(*c*)	$x+6$,	(8)	$x-4$,	(*c*)	$x-5$,
(4)	$x+8$,	(*d*)	$x+5$,	(9)	$x-7$,	(*d*)	$x-12$,
(5)	$x+3$.	(*e*)	$x+7$.	(10)	$x-2$.	(*e*)	$x-3$.

(11)	$x+1$,	(a)	$x-2$,	(16)	$2x+1$,	(a)	$2x+3$,
(12)	$x+7$,	(b)	$x-1$,	(17)	$5x+2$,	(b)	$2x-1$,
(13)	$x+12$,	(c)	$x-5$,	(18)	$x+2$,	(c)	$4x+7$,
(14)	$x+4$,	(d)	$x-9$,	(19)	$4x-7$,	(d)	$3x+5$,
(15)	$x+2$.	(e)	$x-10$.	(20)	$x-8$.	(e)	$7x-1$.

(21)	$a-4$,	(a)	$a+4$,	(26)	$y+2$,	(a)	$y-5$,
(22)	$2a+1$,	(b)	$4a+3$,	(27)	$2y-1$,	(b)	$y+1$,
(23)	$4a-3$,	(c)	$a-7$,	(28)	$y+5$,	(c)	$y-2$,
(24)	$a+7$,	(d)	$a+11$,	(29)	$3y-7$,	(d)	$4y+1$,
(25)	$2a-11$.	(e)	$6a+1$.	(30)	$y-1$.	(e)	$2y+1$.

Ex. 42. PRODUCTS BY INSPECTION.

In numbers 1 to 6 state the degree of each of the expressions to be multiplied together, the degree of the product and the coefficient of x^4 in the product.

(1) $(x+1)(x^4+4x^3+2)$. (2) $(2x^2+6x+1)(4x^3+2x^2+x+3)$.

(3) $(x+4)(x^7+x^6+x^3+1)$. (4) $(x^2+2)(x^4+2x^2+1)$.

(5) $(x^2+2)(x^5+x^3+x+1)$. (6) $(2x^3+x+1)(x^7+x^4+x^2+1)$.

Give the highest power of x, and the lowest (with their coefficients) in

(7) $(x^2+x)(x^3+6x+1)$. (8) $(x^2+2x)(x^6+x^3+x^2-2)$.

(9) $(x^3+1)(x^9+2)$. (10) $(3x+2x^2)(4x^3+5+2x)$.

By picking out the coefficients in order, write down the results of the following multiplications:

(11) $(x+1)(x^4+x^3+x^2+x)$. (12) $(a^2+2a+1)(a^3+1)$.

(13) $(y^2-2y+2)(y^2+2y-2)$. (14) $(2x^2+3x+1)(x^2-x+1)$.

(15) $(3x-5)(x^4-2x^3+x^2+4x-3)$.

(16) $(x^8-x^4+1)(x^8+x^4+1)$.

[Ex. 36 will afford further practice in this method.]

FACTORS.

Ex. 43. *Inspection.*

(1) $ax + a^2$. (2) $ax - 2a^2$. (3) $3x^2 - 6x$.

(4) $8x^2 + 4x$. (5) $xy + 2y$. (6) $x^2 + x$.

(7) $a^3 - a^2$. (8) $3x^3 - 8x^2y$. (9) $2xyz + y^2z^2$.

(10) $ab^2c - 6b^2$. (11) $p^3 - 3p^2$. (12) $l^2m^2 + 2lmn^2$.

(13) $b^4 - 3b^2k^2$. (14) $13a^2 - 39ab$. (15) $51x^2 - 34$.

(16) $ax^2 + 2axy - a^2x$. (17) $7a^2d^2 + 28abcd + 6ad^3$.

(18) $7x^2 + 14x - 35$. (19) $a^2bc + ab^2c + abc^2 + abcd$.

(20) $x^3y^3 + 3x^3y^2z - x^2y^2z^2$. (21) $34x^3y^2 + 2xy^2 - 17x^2y + 6xy$.

Ex. 44. *Grouping.*

(1) $x^2 + ax + bx + ab$. (2) $x^2 - cx + ax - ac$.

(3) $2x^2 + ax + 2bx + ab$. (4) $a^2b^2 - abd + acb - cd$.

(5) $2y^2 + ay - 2by - ab$. (6) $x^2 - 2cx + ax - 2ac$.

(7) $x^2 - ax + 7x - 7a$. (8) $x^2 - 3x - cx + 3c$.

(9) $10 + 2a - 5b - ab$. (10) $3 - abx + b - 3ax$.

(11) $ax + a + b + bx$. (12) $x^2p + a^2q - a^2p - x^2q$.

(13) $x^3 + x^2 + ax + a$. (14) $x^3 + x^2 + x + 1$.

(15) $x^3 - 2x^2 + x - 2$. (16) $a^3 + a^2b + ab^2 + b^3$.

(17) $ln + pl - l^2 - pn$. (18) $a^3 - 3a^2 - 2a + 6$.

(19) $tw - wz - kz + tk$. (20) $a^2b^2x - 3ax^2z^2 - 3axyz^2 + a^2b^2y$.

(21) $bt - ab + a^2 - at$. (22) $2ax + ay + 2bx + bz + az + by$.

(23) $x^5 + x^4 + x^3 + x^2 - x - 1$. (24) $2ax - bz + cz - 2az + bx - cx$.

(25) $a^2xy + 3ax + 2x + ay + 3 + 2ax^2$.

Ex. 45. *Trinomials.*

(1) $x^2 + 8x + 15$. (2) $x^2 + 7x + 6$. (3) $a^2 + 5a + 6$.

(4) $x^2 - 7x + 6$. (5) $b^2 - 3b + 2$. (6) $x^2 + 12x + 35$.

(7) $y^2 - 20y + 100$. (8) $x^2 + 11x + 28$. (9) $x^2 - 13x + 40$.

(10) $x^2 + 20x + 99$. (11) $x^2 + 6x + 9$. (12) $a^2 - 14a + 49$.

(13) $x^2 - 23x + 132$.
(14) $x^2 + 25x + 156$.
(15) $a^2 - 8ab + 15b^2$.
(16) $a^2b^2 + 8ab + 15$.
(17) $l^2 - 16lp + 63p^2$.
(18) $x^2y^2 + 7xyz + 6z^2$.
(19) $8 - 6x + x^2$.
(20) $66 - 17y + y^2$.
(21) $45 + 18p + p^2$.
(22) $x^2 + x - 12$.
(23) $x^2 + x - 72$.
(24) $x^2 - 5x - 84$.
(25) $x^2 + 3x - 18$.
(26) $a^2 + 4a - 77$.
(27) $b^2 - 8b - 9$.
(28) $x^2 - 10x - 24$.
(29) $c^2 - 20c + 36$.
(30) $b^2 - 7b - 78$.
(31) $x^2 - 2x + 1$.
(32) $x^2 - 2x - 3$.
(33) $x^2 - 2x - 120$.
(34) $6 + x - x^2$.
(35) $63 - 2x - x^2$.
(36) $20 - 19x - x^2$.
(37) $20 - 21x + x^2$.
(38) $a^2b^2 - 4abc - 5c^2$.
(39) $x^2 - 2xyz + z^2$.
(40) $x^2 - 21x - 72$.
(41) $64 - 12y - y^2$.
(42) $144 + 7ab - a^2b^2$.
(43) $a^2 + 18ay - 243y^2$.
(44) $c^2 + 24cz - 81z^2$.
(45) $x^2 + xy - 2y^2$.
(46) $x^2 + xy - 156y^2$.
(47) $x^2 + xy - 4970$.
(48) $a^2c^2 + 22ac + 105$.
(49) $a^2c^2 - 22ac - 240$.
(50) $a^2c^2 + 22ac - 363$.

Ex. 46.

Perfect squares (if not, say 'not a square').

(1) $a^2 - 2ax + x^2$. (2) $b^2 + 4b + 4$. (3) $x^2 - 4ax + 4a^2$.
(4) $16x^2 - 8ax + a^2$. (5) $a^2b^2 + 2abc + c^2$.
(6) $a^2b^2 + 2abc + 4c^2$. (7) $49a^2 + 28ab + 4b^2$.
(8) $36x^2 - 2xy + y^2$. (9) $x^2 - 18xy + 81y^2$.
(10) $4l^2 - 4lm + m^2$. (11) $a^2b^2c^2 + 6abcz + 9z^2$.
(12) $a^2b^2 - 12ab + 81$. (13) $4c^2 - 20cd + 25d^2$.
(14) $x^2 - 18xy + 81$. (15) $225a^2b^2x^2 - 30abx + 1$.
(16) $a^2 + \frac{2a}{x} + \frac{1}{x^2}$. (17) $\frac{a^2}{b^2} - \frac{2ax}{b} + x^2$. (18) $\frac{x^2}{4} - xy + \frac{1}{y^2}$

What must be added in each of the following cases to make the result a perfect square, and of what is the result a square?

(19) x^2+4x. (20) x^2-4ax. (21) x^2+18bx.

(22) y^2-12y. (23) x^2+8x. (24) a^2-6a.

(25) b^2-12ab. (26) x^2+5x. (27) y^2-3y.

(28) x^2+x. (29) $4x^2+4x$. (30) $9y^2-18y$.

(31) $16a^2-24ab$. (32) a^2b^2+3abc. (33) l^2m^2-lm.

(34) $4y^2+2y$. (35) $49a^2b^2+abx$. (36) $9x^2y^2z^2-21abxyz$.

Ex. 47. *Difference of Two Squares.*

(1) a^2-9. (2) b^2-4. (3) c^2x^2-81.

(4) $49-x^2$. (5) $64-a^2x^2$. (6) $81-4a^6x^6$.

(7) $400-x^2$. (8) $x^2-9a^2y^2$. (9) $100a^2-49$.

(10) $81y^2-b^2c^2$. (11) $121-l^2m^2$. (12) $169-4y^2$.

(13) x^4-9y^4. (14) $16a^4-25b^4$. (15) $64a^6-c^6$.

(16) $225-z^{10}$. (17) $121a^2b^2-4$. (18) $\overline{a+x}|^2-1$.

(19) $\overline{y+z}|^2-4x^2$. (20) $\overline{a-b}|^2-c^2$. (21) $\overline{x+y}|^2-\overline{a+b}|^2$.

(22) $(l+m)^2-(k+p)^2$. (23) $(x+y)^2-(a-b)^2$.

(24) $(x-y)^2-(a+b)^2$. (25) $(x-y)^2-(a-b)^2$.

(26) $a^2-(b-c)^2$. (27) $4z^2-(x-y)^2$.

(28) $64l^2-(m-8)^2$. (29) $(x+y)^2-(x-y)^2$.

(30) $(2x+3y)^2-(x+2y)^2$. (31) $4x^2-(x-y)^2$.

(32) $(6x+4y)^2-(4x+6y)^2$. (33) $(3a+4x)^2-(3a-4x)^2$.

(34) $(2x-y+3z)^2-(2x+y-3z)^2$. (35) 725^2-275^2.

(36) 1024^2-1023^2. (37) 63^2-53^2.

(38) 1176^2-1170^2. (39) $(1827)^2-927^2$.

(40) 1002^2-4. (41) $\left(x+\frac{1}{2}\right)^2-\frac{1}{4}$.

(42) $\left(x-\frac{1}{3}\right)^2-\frac{1}{9}$. (43) $\left(a+\frac{1}{2}\right)^2-1$.

(44) $\left(cd+\frac{3}{4}\right)^2-\frac{9}{16}$. (45) $a^2+2ab+b^2-c^2$.

(46) $a^2 - 2ac + c^2 - b^2$.
(47) $x^2 - y^2 - 2yz - z^2$.
(48) $16a^2 - b^2 - 4bc - 4c^2$.
(49) $x^2 - y^2 + 2yz - z^2$.
(50) $x^2 - y^2 + 4yz - 4z^2$.
(51) $9a^2 - 6ab + b^2 - 81x^2$.
(52) $a^2 - 2ab + b^2 - x^2 + 2cx - c^2$.
(53) $a^2 - b^2 + c^2 - 2ac$.
(54) $9x^2 + 16z^2 - t^2 - 24xz$.
(55) $x^2 - c^2 - b^2 + y^2 - 2xy + 2bc$.
(56) $a^4 + x^4 - a^2 - 9 - 2a^2x^2 + 6a$.
(57) $x^4 + x^2y^2 + y^4$.
(58) $a^4 + 9a^2b^2 + 81b^4$.
(59) $x^4 + 121y^4 - 38x^2y^2$.

Ex. 48. *Trinomials (continued).*

(1) $2x^2 - x - 1$.
(2) $2x^2 + x - 1$.
(3) $2x^2 + 3x - 2$.
(4) $3y^2 - 4y - 4$.
(5) $5y^2 + 26y + 5$.
(6) $3 - 7y + 2y^2$.
(7) $3 - 5y - 2y^2$.
(8) $3 + 8y + 4y^2$.
(9) $6a^2b^2 + ab - 5$.
(10) $15x + 2 - 8x^2$.
(11) $9 + 6x - 8x^2$.
(12) $6a^2 - 7a - 3$.
(13) $4b^2 + 23b + 15$.
(14) $2 - 3a - 2a^2$.
(15) $6y^2 + 5y - 21$.
(16) $5c^2 - 11c + 2$.
(17) $30a^2 - 37a + 10$.
(18) $15 - 8y - 12y^2$.
(19) $15 - 28y + 12y^2$.
(20) $72 - 17x - 72x^2$.
(21) $2a^2 + 3ab - 20b^2$.
(22) $3x^2 - 31x + 56$.
(23) $9a^2 - 64ac + 60c^2$.
(24) $72 - 219x + 72x^2$.

Ex. 49. *Sum or Difference of Two Cubes.*

(1) $x^3 + y^3$.
(2) $x^3 + 8a^3$.
(3) $b^3 - c^3$.
(4) $x^6 - y^3$.
(5) $27a^3 + 64b^3$.
(6) $8x^3 - y^3$.
(7) $a^3 - 1000b^3$.
(8) $x^3 - 8$.
(9) $(a+b)^3 - c^3$.
(10) $(x+y)^3 + z^3$.
(11) $(2zx)^3 - 27$.
(12) $27 - y^3$.
(13) $l^3m^3 + (l^2 + m^2)^3$.
(14) $64 - x^3$.
(15) $8a^3y^3 + 27z^3$.
(16) $\frac{1}{x^3} + \frac{1}{y^3}$.
(17) $a^3 + \frac{8}{b^3}$.
(18) $\frac{a^6b^6}{27} - \frac{27}{c^3}$.
(19) $216 - x^3y^3$.
(20) $\frac{64}{a^3} - \frac{27}{b^3}$.

Ex. 50. SOLUTION OF EQUATIONS BY FACTORS.

Solve the following equations:

(1) $(x-6)(x-2)=0$.
(2) $(x-5)(x-4)=0$.
(3) $(2x-1)(x-3)=0$.
(4) $(x+2)(x-5)=0$.
(5) $(x-3)(x+3)=0$.
(6) $(2x+9)(x-7)=0$.
(7) $(x-1)(x+2)(2x-1)=0$.
(8) $(x-1)^2(x+2)=0$.
(9) $(x-4)(2x-7)^2=0$.
(10) $(x-a)(bx+c)=0$.
(11) $x(x+3)(x-2)=0$.
(12) $x^2(x+1)=0$.
(13) $x^2-5x+6=0$.
(14) $x^2+4x+3=0$.
(15) $x^2-2x-35=0$.
(16) $x^2+5x=6$.
(17) $2x^2+x-1=0$.
(18) $x^2-21x+104=0$.
(19) $15x^2=x+2$.
(20) $x^2-3ax+2a^2=0$.
(21) $3x^2=10+13x$.
(22) $x^2-144=0$.
(23) $x^2-2x=0$.
(24) $2x^2+17x=0$.
(25) $x^3-x=0$.
(26) $4x^2-b^2=0$.
(27) $x^2-20=8x$.
(28) $x^3=4x$.
(29) $7x^3+6x^2-x=0$.
(30) $ax^2-a^2x-bx+ab$.
(31) $5x^2-24px=5p^2$.
(32) $a^2x^2-a^4x-a^2+x$.

Ex. 51. CONSTRUCTION OF EQUATIONS WITH GIVEN SOLUTIONS.

What are the equations, whose solutions are the following? (All brackets should be removed in at least some cases and the results tested by substitution.)

(1) 5, 1.
(2) 3, 4.
(3) $2, \frac{1}{2}$.
(4) $5, -5$.
(5) $3, -7$.
(6) $-2, -\frac{1}{4}$.
(7) 3, 1, 0.
(8) $1, -1, 2$.
(9) $-\frac{1}{2}, \frac{2}{3}$.
(10) $a, -a$.
(11) $2b, -b$.
(12) $-p, -7p$.
(13) $a+b, a-b$.
(14) $a, \frac{1}{a}$.
(15) $-2c, \frac{2}{c}$.
(16) a, b, c.
(17) $-a, -b, -c$.
(18) $a, a, -2a$.

Ex. 52. H.C.F. AND L.C.M. BY FACTORS.

(*Answers to be given in factors.*)

(1) x^2-y^2, $x^2-2xy+y^2$.
(2) a^2-b^2, $a+b$.
(3) $a^2+2ab+b^2$, a^2-b^2, $a+b$.
(4) x^2+4x+3, x^2+3x.
(5) x^2-2x+1, x^2-1.
(6) $a^2+7ab+12b^2$, $a^2+3ab-4b^2$, a^2-16b^2.
(7) x^2-a^2, $(x+a)^2$, $(x-a)^2$.
(8) $a^2-2ab+b^2$, a^3-b^3.
(9) x^3+y^3, x^2-y^2, $x^2+2xy+y^2$.
(10) x^3y-xy^3, x^3y+xy^3, x^5y-xy^5.

Ex. 53. SIMPLIFICATION OF FRACTIONS.

(1) $\dfrac{x+3}{x+5}\times\dfrac{x+1}{x+3}\times\dfrac{x+5}{x+4}\times\dfrac{x}{x+1}$.
(2) $\dfrac{x+2}{x-2}\div\dfrac{x+3}{x-2}\times\dfrac{x+3}{x+2}$.
(3) $\dfrac{x^2-y^2}{x}\times\dfrac{2xy}{x^2+2xy+y^2}\times\dfrac{x-y}{2y}$.
(4) $\dfrac{a^2-b^2}{a}\times\dfrac{1}{a+b}\times\dfrac{a}{a-b}$.
(5) $\dfrac{15x-30}{4x}\times\dfrac{3x^2}{5x-10}$.
(6) $\dfrac{(x-1)^2}{y^3}\times\dfrac{(x+1)y^2}{x-1}$.
(7) $\dfrac{a^2-x^2}{a}\times\dfrac{a^2+x^2}{ax}\times\dfrac{a^2}{a^4-x^4}$.
(8) $\dfrac{a^2+2ab}{b^2}\times\dfrac{b^3}{a^2-b^2}\times\dfrac{a-b}{a^2+3ab+2b^2}\times(a+b)^2$.
(9) $\dfrac{a+x}{(m+n)^2}\div\dfrac{m+n}{a-x}\times\dfrac{m^2-n^2}{a^2-x^2}$.
(10) $\dfrac{a+x}{(m+n)^3}\times\dfrac{x^2-y^2}{12}\div\left\{\dfrac{m-n}{(m+n)^2}\times\dfrac{x+y}{6(m^2-n^2)}\right\}$.

Ex. 54. QUADRATIC EQUATIONS.

Oral. [If there are no real roots, say 'no real roots.']

(1) $x^2=81$.
(2) $x^2=169$.
(3) $5x^2=125$.
(4) $7x^2=112$.
(5) $4x^2-11=25$.
(6) $9x^2-35=100$.
(7) $x^2+5=12$.
(8) $5x^2+61=37$.
(9) $(x-4)^2=64$.
(10) $(x+3)^2=49$.
(11) $(2x-1)^2=1$.
(12) $x^2+12x+36=81$.
(13) $4x^2-20x+25=100$.
(14) $x^2+7x+\dfrac{49}{4}=\dfrac{1}{4}$.
(15) $x^2+7x=-12$.

Ex. 55. [*Nos. 1 to 20 by completing square.*]

Solve the equations:

(1) $x^2 - 40x = 441$.
(2) $x^2 + 28x = 204$.
(3) $x^2 - 72x = 73$.
(4) $x^2 + 32x - 33 = 0$.
(5) $x^2 - 34x + 120 = 0$.
(6) $x^2 - 88 = 18x$.
(7) $x^2 - 2x = 130$.
(8) $x^2 = 20x - 36$.
(9) $x^2 - 31x + 228 = 0$.
(10) $x^2 + 26x + 48 = 0$.
(11) $x^2 - x - 8 = 0$.
(12) $x^2 + 12x + 100 = 0$.
(13) $x^2 - 23x = -126$.
(14) $x^2 + 7x = 30$.
(15) $2x^2 + 7x + 4 = 0$.
(16) $x^2 - 3x - 180 = 0$.
(17) $x^2 + x + 41 = 0$.
(18) $x^2 - 56 = -x$.
(19) $3x^2 - 2x + 5 = 0$.
(20) $3x^2 + 2x - 5 = 0$.

(21) Find a from $a^2 - a - 20 = 0$.

(22) Find $\frac{x}{y}$ from $x^2 - 7xy + 12y^2 = 0$.

(23) Find $\frac{x}{y}$ from $x^2 - 4xy - 21y^2 = 0$.

(24) Find $\frac{x}{a}$ from $x^2 + 2ax = 3a^2$.

(25) Find $\frac{a}{b}$ from $a^2 + 8b^2 = 6ab$.

(26) Find $\frac{x}{y}$ from $2x^2 - 5xy - 3y^2 = 0$.

(27) Find y from $(y+1)(y+2) - 22 = 2(y^2 - 100)$.

Solve for x,

(28) $\frac{2x-5}{x-1} = \frac{3x+1}{x+9}$.
(29) $\frac{x+2}{5x-1} = \frac{x-3}{4x+1}$.
(30) $x^2 = 3 + (4x+1)^2$.
(31) $\frac{x+8}{x-8} = \frac{3x-5}{x+5}$.

(32) Find $\frac{x}{y}$ from $\frac{2x-5y}{y} = \frac{2x+4y}{x}$.

(33) Find $\frac{x}{y}$ from $\frac{x+y}{x} = \frac{7x+5y}{x-y}$.

(34) Find $\frac{x}{y}$ from $\frac{2x+y}{x} = \frac{3y+7x}{5y-x}$.

(35) Find $\frac{x}{y}$ from $\frac{x-y}{x+y} = \frac{4(x-y)}{2x+y}$.

From the formula $s = ut - 16t^2$ find t,

(36) Given $s = 6$, $u = 50$.

(37) Given $s = 256$, $u = 128$.

(38) Given $s = 100$, $u = 160$.

(39) Given $x^2 + y^2 = 4$,

(40) Given $x^2 + y^2 = 6x + 4y$,

} find { (α) the values of y if $x = 1$,
(β) the values of x if $y = 1$.

Ex. 56. FACTORS OF QUADRATICS: GENERAL CASE.

[Give all real factors, whether containing surds or not; the general method of completing the square should be used in Exs. 1 to 16. If there are no real factors, say 'no real factors.']

(1) $x^2 - 8x + 9$.

(2) $x^2 + 4x - 1$.

(3) $x^2 + 2x + 27$.

(4) $2x^2 - 9x - 56$.

(5) $15x^2 - 11x - 14$.

(6) $4x^2 + 4x - 2$.

(7) $9x^2 + 30x + 14$.

(8) $9x^2 + 30x + 36$.

(9) $x^2 + x - 1$.

(10) $x^2 - x - 1$.

(11) $x^2 - x + 1$.

(12) $4x^2 + 16xy + 7y^2$.

(13) $x^2 - xy - 3y^2$.

(14) $25x^2 + 20axy - 17a^2y^2$.

(15) $x^2 + 3xy + 3y^2$.

(16) $4x^2 - 44x + 119$.

(17) $x^2 - 3$.

(18) $x^4 - 4a^4$.

(19) $x^4 + 9a^4$.

(20) $x^3 + 10$.

(21) $x^3 - 4a^3$.

(22) $x^6 - 64b^3$.

(23) $9x^4 - 1$.

(24) $2x^4 - 7x^2 + 3$.

Ex. 57. SIMULTANEOUS QUADRATICS.

(1) $x^2 = y + 3$, $y = x + 3$.

(2) $3x = y + 5$, $y^2 + 3x = 17$.

(3) $x - 3y = 7$, $x^2 - 4y^2 + 12y = 9$.

(4) $2x - y + 3 = 0$, $y^2 - x^2 + 2x = 1$.

(5) $2x = 3y + 1$, $x^2 = 9y - 2$.

(6) $3y - 5x = 6$, $2y^2 = 5x + 5$.

(7) $2x - 3y = 1$, $3xy + 6x - 42y = 84$.

(8) $3x + y = 12$, $4x^2 - y^2 = 5x$.

(9) $3x + 2y = 13$, $x^2 + 2y^2 - 4x = 7$.

(10) $13y - 9x = 1$, $x^2 - 2xy - 2y^2 = 22$.

(11) $\left.\begin{aligned} x+y&=0,\\ 3x^2+4xy&=8y-33. \end{aligned}\right\}$ (12) $\left.\begin{aligned} 2y-9x&=1,\\ x^2+y^2+x+3y&=250. \end{aligned}\right\}$

(13) $\left.\begin{aligned} 2x-y&=7,\\ x^2-2y^2&=7x-14. \end{aligned}\right\}$ (14) $\left.\begin{aligned} 2x-y&=7,\\ x^2-2y^2&=7x+14. \end{aligned}\right\}$

(15) $\left.\begin{aligned} 2x-y&=7,\\ x^2-2y^2&=7x-17. \end{aligned}\right\}$ (16) $\left.\begin{aligned} 3x&=4y+80,\\ xy&=56y. \end{aligned}\right\}$

(17) $\left.\begin{aligned} x+y&=11,\\ xy&=18. \end{aligned}\right\}$ (18) $\left.\begin{aligned} x-y&=12,\\ xy&=13. \end{aligned}\right\}$

(19) $\left.\begin{aligned} x-y&=1,\\ xy&=56. \end{aligned}\right\}$ (20) $\left.\begin{aligned} x+y&=5,\\ xy+14&=0. \end{aligned}\right\}$

(21) $\left.\begin{aligned} x+y&=15,\\ xy&=12. \end{aligned}\right\}$ (22) $\left.\begin{aligned} x+y&=20,\\ xy&=102. \end{aligned}\right\}$

(23) $\left.\begin{aligned} x-y+2z&=0,\\ -2x+y+2z&=0,\\ x^2+y^2-5z^2&=47. \end{aligned}\right\}$ (24) $\left.\begin{aligned} x+y&=2,\\ 3x+y-z&=0,\\ x^2+y^2+z^2&=10. \end{aligned}\right\}$

(25) Show that the equations

$$\left.\begin{aligned} 2x^2&=2y-x\\ 4y^2-13x^2&=4x-8 \end{aligned}\right\}$$

lead to an equation for x of the fourth degree: being given that $x=\pm 1$ are roots, complete the solution.

(26) Show that the equations

$$\left.\begin{aligned} y^2+3&=x-y\\ x^2-12y^2&=12y+9 \end{aligned}\right\}$$

lead to an equation for y of the fourth degree: being given that $y=2$ is a root, complete the solution.

[Examples of the most important cases where two equations of the second degree in two unknowns can be solved by quadratics, will be found under 'Miscellaneous Artifices.']

Ex. 58. PROBLEMS.

(1) Find a number such that its square is four times the product of the number and twelve.

(2) Find a number such that its square is four times the sum of the number and twenty-four.

(3) The sum of the squares of two consecutive numbers is 2813: find them.

(4) The sum of the squares of two consecutive even numbers is 7444: find them.

(5) The sum of the squares of two consecutive odd numbers is 3202: find them.

(6) The sum of the reciprocals of two consecutive even numbers is $\frac{9}{40}$: find them.

(7) The difference of the reciprocals of two numbers which differ by 3 is $\frac{1}{60}$: find the numbers.

(8) The difference between the square of twice a number and the square of half it is 1815: find the number.

(9) The difference between the square of a number and the square of three-fifths of it is 2704: find the number.

(10) What are the sides of a rectangle whose area is 91 sq. ft. and whose perimeter is 40 ft.?

(11) What are the sides of a rectangle whose perimeter is 74 ft. and whose area is 336 sq. ft.?

(12) The length of a rectangle is half as much again as its breadth. If its area is 1176 sq. ft., what is its length?

Ex. 59. APPLICATIONS TO GEOMETRY.

A straight line AB one foot long is divided at C so as to satisfy one of the following conditions. Find in each case the length of AC:

(1) If $AC^2 = AB \cdot BC$.

(2) If $AC^2 = 2AB \cdot BC$.

(3) If $2AC^2 = 3AB \cdot BC$.

(4) If $AC^2 = 2BC^2$.

(5) If $AC^2 = AB^2 - 2BC^2$.

(6) If $AC^2 - BC^2 = \frac{1}{4}$ sq. ft.

If c is the length of the hypotenuse of a right-angled triangle, and a, b, the lengths of the other two sides:

(7) Find c if $a=20$, $b=48$.

(8) Find a if $c=39$, $b=15$.

(9) Find a and b if $c=50$, $ab=1200$.

(10) Find a and c if $b=7$ and the perimeter of the triangle is 56.

(11) Find a and b if $c=40$ and the perimeter of the triangle is 85.

Ex. 60. QUADRATIC GRAPHS.

[It is intended that the graph in (1) be drawn carefully say between $x=\pm 3$, but those in Nos. (2) to (8) be not drawn in more detail than is necessary to get a clear idea of the general shape.]

Draw the graphs of:

(1) $y=x^2$. (2) $y=-x^2$. (3) $y=2x^2$.

(4) $y=x^2+2{\cdot}5$. (5) $y=(x-1)^2$. (6) $y=(x+2)^2+1$.

(7) $y=x^2+4x+6$. (8) $y=x^2-3x+1$.

(9) Write out a general statement of the difference between the graphs of $y=x^2$ and of $y=\pm a\{(x-b)^2+c\}$.

(10) Draw the graph of $y=x^2-3x+\frac{9}{4}$ and hence find approximately the roots of $x^2-3x+1=0$,
and of $x^2-3x-2=0$.
Also for the equation $x^2-3x+c=0$, point out what values of c will make the roots real, equal, imaginary.

(11) Draw the graph of $y=x^2+6x+8$ and use it to solve approximately $x^2+6x+6=0$.

(12) Draw the graph of $y=x^2-4x+3$ and show that y is positive except when x lies between the roots of $x^2-4x+3=0$. How can this be proved without using the graph?

(13) Draw the graph of $y=x^2+x+1$ and use it to solve approximately $x^2+x-1=0$.

(14) By drawing $y=x^2$ and $y=2x+1$, find the roots of $x^2-2x+1=0$.

(15) By drawing $y=x^2$ and finding where it is cut by certain straight lines, solve the equations:

(i) $x^2+x-1=0$;
(ii) $2x^2-2x-1=0$;
(iii) $2x^2-2x-3=0$.

Ex. 61. MISCELLANEOUS.

(1) Prove that
$$(x^2-y^2)(z^2-w^2)=(xz+yw)^2-(xw+yz)^2$$
$$=(xz-yw)^2-(xw-yz)^2.$$

(2) Find the continued product
$$(a+b+c)(a+b-c)(c+a-b)(b+c-a).$$

(3) Multiply out
$$(x^2+2xy+y^2)(x^2-2xy+y^2)(x^2-y^2).$$

(4) Find the square of $a-\frac{b}{2}+\frac{c}{3}-\frac{d}{4}$.

(5) Find the coefficient of x^5 in the product
$$(x^5-x^4+3x^2-2x+7)(x^5-4x^3+5x^2+5).$$

---

(6) Find four factors of
(i) x^8-1; (ii) $3bx^4-3by^4$; (iii) x^6-z^6.

(7) Factorize
(i) $3a^5-48ac^8$; (ii) $x^4+x^3-x^2-x$; (iii) $a^{4p}-b^{4q}$.

(8) Put into factors
(i) $4(ab+cd)^2-(a^2+b^2-c^2-d^2)^2$; (ii) $81x^4-y^4$.

(9) Factorize
(i) $64x^6-y^6$; (ii) $x^4-10x^2y^2+9y^4$; (iii) x^4+y^4.

(10) Factorize
(i) $(2ab)^2-(a^2+b^2-c^2)^2$; (ii) $6x^2+xy-12y^2$.

---

(11) Solve the equations
(i) $(x-1)=2(x-\frac{3}{2})$;
(ii) $\frac{1}{2}(x-1)-\frac{x-5}{9}+5.\frac{x-1}{6}=0$;
(iii) $3x+2y=1$, $2x^2-xy=4$.

(12) Solve the equations (i) $\frac{3x-2}{2x-5}-\frac{2x-5}{3x-2}=\frac{8}{3}$;

(ii) $\frac{x-1}{x-5}=\frac{x-2}{x-4}$; (iii) $\left.\begin{aligned} x+2y&=4,\\ 2x^2-y^2&=7. \end{aligned}\right\}$

(13) For the following expressions, each of which is of the form ax^2+bx+c, calculate the value of the quantity b^2-4ac:

(i) $2x^2-7x+12$; (ii) $2x^2-7x-12$;
(iii) $3x^2-8x+5$; (iv) $9x^2-12x+4$.

(14) Form the equations whose roots are

(i) 7, 3; (ii) 8, -3; (iii) 5, $\frac{1}{2}$; (iv) -6, -4;
(v) a, b; (vi) $a+b$, $a-b$; (vii) $a-b$, $b-a$.

(15) Prove that if x, y satisfy the equations

$$\left.\begin{aligned} x^2+y^2-8&=0\\ 2x+y-6&=0 \end{aligned}\right\} \quad \text{.....(i)}$$

then they must also satisfy the equations

$$\left.\begin{aligned} x^2+y^2-8+(2x+y-6)(x+2y-6)&=0\\ x^2+y^2-8-(2x+y-6)(x+2y-6)&=0 \end{aligned}\right\} \quad \text{.....(ii)}$$

Find values of x, y to satisfy (ii), but not to satisfy (i).

---

(16) If C divides AB so that $AC^2=AB.BC$, prove that

$$AB^2+BC^2=3AC^2.$$

(17) The sum of the radii of two circles is 9 inches, and the sum of their areas is 198 square inches. Find the radii of the circles. (Area $=\frac{22}{7}.\overline{\text{radius}}^2$).

(18) How can a debt of £4. 5s. be paid in half-crowns and shillings, so that the number of half-crowns is thrice that of the shillings?

(19) In a railway journey of 90 miles an increase of 5 miles an hour in the rate diminishes the time taken by 15 minutes. What was the rate?

(20) The price of seats to view a procession was raised by 10s. a-piece, and in consequence 7 less than before were sold for 40 guineas. What was the original price?

[Questions 21-25 are intended to be answered from graphical considerations. In drawing the graphs any convenient values may be taken for a, b, c which are supposed positive.]

(21) Why must $ax^2+bx-c=0$ have real roots?

(22) If $ax^2+bx+c=0$ has no real roots, show that if k be chosen so that $ax^2+bx+c=k$ has equal roots, then k is the least possible value of ax^2+bx+c.

(23) Describe in the general terms the difference between the graphs of $y_1=ax^2+bx+c$
and $y_2=ax^2-bx+c$.
Show that they are the same size, and that if y_1 can be 0 so can y_2.

(24) Since x and $x+k$ are necessarily different, how do you account for the fact that the graphs of $y_1=x^2$ and $y_2=(x+k)^2$ intersect?

(25) Is the difference of the roots of $ax^2+bx+c=k$ (where k is positive) greater or smaller than the difference of the roots of $ax^2+bx+c=0$? How could you prove your result without appealing to the graph?

Ex. 62. SYMBOLIC REPRESENTATION.

Oral.

Percentages.

What is

(1) x % of £2. (2) y % of £3. (3) 4 % of £x.

(4) a % of £b. (5) p % of £q.

(6) Interest each year on £200 at x %.

(7) Interest each year on £x at 5 %.

(8) Interest each year on £x at y %.

(9) Simple interest on £z at x % for 2 years.

(10) Simple interest on £P at R % for Y years.

(11) The selling price if the cost is £300 and the gain x %.

(12) The selling price if the cost is £a and the gain x %.

(13) The selling price of goods costing £b and sold at a loss of 5 %.

(14) The amount of £600 lent at x % compound interest for 2 years.

(15) The amount after 3 years of £a lent at y % compound interest.

(16) The amount after Y years of £P lent at R % compound interest.

(17) The income obtained from £500 of x % stock.

(18) The income obtained from £a of b % stock.

(19) The amount of a stock whose price is 94 which can be obtained for £x cash.

(20) The amount of a stock whose price is 116 which can be obtained for £y cash.

(21) The amount of a stock whose price is x which can be obtained for £a cash.

(22) The income obtained by investing £c cash in 4 % stock at 108.

(23) The income obtained by investing £780 cash in an x % stock at 97.

(24) The income obtained by investing £a cash in a 3 % stock at x.

(25) The income obtained by investing £c cash in an R % stock, the price being x.

(26) The difference in the incomes obtained by investing £P in 5 % stock at 112 or in 2 % stock at 81.

(27) The difference in the incomes obtained by investing £7500 in x % stock at 114 or in y % stock at 90.

(28) The difference in the incomes obtained by investing £P in 4 % stock at x or in 3 % stock at y.

Areas, etc.

What is the area of a rectangle whose dimensions are

(29) length a feet, breadth b feet;

(30) length x yards, breadth y feet;

(31) length x yards, breadth y inches;

(32) length double breadth, which is a feet;

(33) length half as much again as breadth, which is x feet;

(34) length 10 feet more than breadth, which is x feet;

(35) length a feet more than breadth, which is y feet.

(36) If the length and breadth are x and y feet, what is the perimeter?

(37) If the perimeter is z feet and the length a feet, what is the breadth?

(38) If the area is x sq. feet and the breadth b feet, what is the length?

(39) If the perimeter is a feet and the breadth b feet, what is the area?

If a room is L feet long, B feet broad, H feet high, what is

(40) the area of the floor? (41) the perimeter?

(42) the area of the ceiling? (43) the total volume?

(44) the area of the four walls?

(45) the area of each end wall?

If a carpet or strip or paper is

(46) x feet broad, (47) y inches broad, } what is the area of { (α) a yard of it? (β) a foot of it?

(48) What area is covered by n yards of carpet z inches broad?

(49) How many yards of carpet, b feet wide, are required for the floor of a room x feet long, y feet wide?

(50) How many feet of carpet x inches wide, are required for the floor of a room a feet long, b feet wide?

If a room is L feet long, B feet broad, H feet high, find

(51) the number of yards of carpet a feet wide required for the floor;

(52) the area of the walls;

(53) the number of feet of paper, x feet wide, required to paper the walls.

(54) The three dimensions of a box are a feet, b feet, c feet, what is its volume?

(55) A box has external dimensions x, y, z inches, and is made of wood 1 inch thick (lid included). What is its inside volume?

(56) What is the area of a circle radius a feet?

(57) What is the area of a circle diameter b feet?

(58) What is the area between the circumferences of two concentric circles of radii x feet and $2x$ feet?

A circular well has radius x feet and depth d feet,

(59) what is the area of its bottom?

(60) what is the area of its side wall?

(61) if water stood 6 feet deep in it what volume would that be?

(62) What is the volume of a pencil 6 inches long and $\frac{1}{4}$ inch in diameter?

Miscellaneous.

(63) At 30 miles an hour,

(i) how long for a miles? (ii) how far in x hours?

(64) At x miles an hour,

(i) how far in 7 hours? (ii) how far in t minutes?

(iii) how long for 20 miles? (iv) how long for a miles?

(65) At p feet per second,

(i) how many yards per minute?

(ii) how many seconds for one yard?

(iii) how long for a feet? (iv) how far in b minutes?

(66) At y miles an hour,

(i) how long for 8 miles? (ii) how far in 8 hours?

(iii) if the pace is decreased 2 miles per hour, how long for 8 miles?

(iv) If the pace is doubled, by how much is the time for 20 miles decreased?

(67) Buying at x for a penny,

(i) how many for 6d.? (ii) what is cost of 1?

(iii) what is cost of z?

(68) Buying at z pence per score,
(i) how many for 1d.? (ii) how many for £1?
(iii) what is cost of x?

(69) Selling at 3 for 5d.,
(i) what is the cost of x? (ii) how many for y pence?
(iii) how many for z shillings?

(70) Selling at x for a pence,
(i) how many for 1d.? (ii) what is the cost of 15?
(iii) what is cost of z dozen? (iv) how many for £1?

(71) If $\frac{x}{y}$ be a fraction, what is the result of
(i) adding 1 to the numerator?
(ii) taking 2 from the denominator?
(iii) adding 1 to both numerator and denominator?
(iv) adding 1 to the fraction?

(72) If $\frac{p}{q}$ be a fraction, what is the result of
(i) doubling it? (ii) adding 2 to it?
(iii) adding 4 to the numerator and taking 3 from the denominator?
(iv) subtracting $\frac{1}{2}$ from it?

(73) To x pints of milk, y pints of water are added,
(i) how much water in a quart of the mixture?
(ii) what fraction of the whole is milk?
(iii) if the amount of milk was doubled, what fraction of the whole would then be water?

(74) In blending tea, x lbs. of (a kind) A is mixed with y lbs of B.
(i) What fraction of the whole is A?
(ii) How much of A is there in 1 lb. of the mixture?
(iii) How much of B is there in z lbs. of the mixture?
(iv) What is the total amount if there is 2 lbs. of B in it?

(75) If A takes x days to do a piece of work and B takes y days, what fraction of the work (i) does A do in a day; (ii) do they do in a day if working together; (iii) what fraction

is left when A has worked 6 days; (iv) how long would B take over this fraction?

(76) A cistern can be filled by a pipe A in x hours and emptied by another pipe B in y hours ($y<x$).

(i) If A has been running for 3 hours how much of the cistern has been filled, and (ii) how much remains to be filled?

(iii) If A and B are turned on together what fraction of the whole is emptied in 1 hour, and (iv) how long would it take to empty the whole?

---

(77) In $\triangle ABC$, figure 1, the angle C is a right angle.

(i) If AB is x, AC is y, what is BC?

(ii) If BC is a, CA is b, what is AB?

(iii) If AC is x, $AB-AC$ is y, what is BC?

(78) In figure 2, O is the centre, ON is perpendicular to AB.

(i) If AB is x, what is AN?

(ii) If AB is x, and ON is p, what is AO?

(iii) If AO is r, and ON is p, what is AB?

(iv) If AO is r, and AB is x, what is ON?

(79) In figure 3, O is the centre; what relations hold

(i) between $AP.PB$ and $DP.PC$?

(ii) between $AP.PB$, OC, and OP?

(80) In figure 4, C is the centre and ON a tangent; what relations hold

(i) between OC, CN, ON?

(ii) between $OT.OS$ and ON?

(iii) between $OT.OS$, OC, and CN?

Ex. 63. APPLICATIONS TO ARITHMETIC.

[Numbers 1 to 9 are examples where a quasi-algebraic method is perhaps easier than a purely arithmetical one. Only a few are given as similar ones will be found in any arithmetic.]

(1) What number gives $6\frac{2}{5}$ as result when divided by $4\frac{3}{4}$?

(2) What is the number of which 1·2 per cent. is 48?

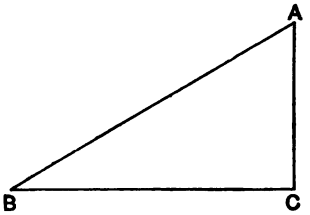


Fig. 1.

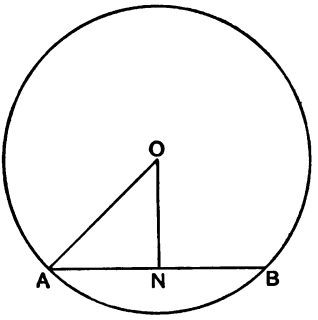


Fig. 2.

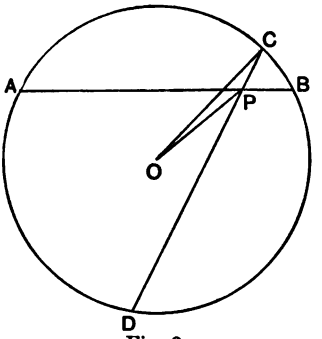


Fig. 3.

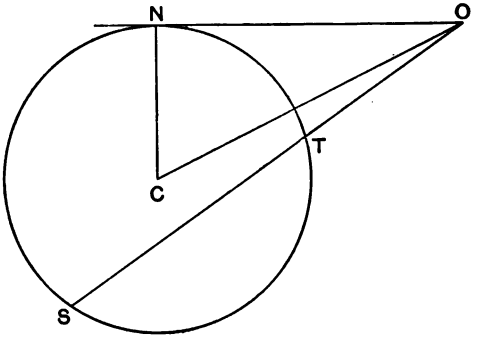


Fig. 4.

(3) What sum of money has £78 as its simple interest for 3 years at 4 %.

(4) If the simple interest on £324 at $2\frac{1}{2}$ % be £36. 10s., what is the number of years?

(5) What was the cost of goods sold for £168 at a gain of 12 per cent.?

(6) On goods sold for £234 there was a loss of 10 %: find their cost.

(7) Investing £390 in a stock at 104 gives an income of £15 a year, what is the rate per cent.?

(8) What is the breadth of the carpet, if 72 yards of it cover a floor 27 feet long by 16 feet broad?

(9) What is the breadth of the paper, if 48 yards of it are required to paper the walls of a room 18 feet square by 12 feet high?

Ex. 64. APPLICATIONS TO GEOMETRY.

(1) In a circle of radius 2·5 inches, a chord 4·8 inches long is placed: find the length of the perpendicular to this chord from the centre.

(2) In a circle of radius 3 inches, a chord 2 inches long is placed: find the length of the perpendicular to this chord from the centre.

(3) A chord 3 inches long is placed in a circle of radius 2 inches: find the distances from the centre to the points of trisection of the chord.

(4) There are two concentric circles whose radii are 3 inches and 5 inches respectively: find the lengths of chords of the larger circle which touch the smaller.

(5) In a circle of radius 1·5 inches, two chords cut having the product of the segments formed equal to 2: find how far their point of intersection is distant from the centre.

(6) Two chords of a circle intersect The length of one is 2·8 inches and the segments of the other are 1·2 inches and 2·8 inches: find the lengths of the segments of the first.

(7) Two chords of a circle intersect. The segments of one of them are 2 inches and 1·5 inches. If the other one is bisected at the point of section find its length.

(8) A line AB is 2 ft. long: find the distance from A of a point P

(i) so that $PA = 2PB$, } { (α) if P lies in AB,
(ii) so that $PA^2 = 2PB^2$. } { (β) if P lies in AB produced.

(9) C is the centre of a circle radius 3 inches. At A a point on the circle a tangent is drawn: find (i) a point P on the tangent such that the part of PC outside the circle is two-thirds of AP; (ii) a point Q on the tangent such that $AQ = \frac{4}{5} CQ$.

(10) It is known that the sides of a right angled triangle are represented by 3 consecutive whole numbers: find them.

(11) Find the area of a right angled isosceles triangle,
(i) if the hypotenuse is 50 ft. long.
(ii) if the perimeter of the triangle is 200 ft.

(12) The length of a rectangle is 10 ft. more than its breadth and its area is 9776 sq. ft.: find the lengths of its sides

(13) A rectangular field is 1 acre in area and its perimeter is 302 yds.: find its length and breadth.

(14) The diagonal of a rectangle is 17 yds. and its area is 120 sq. yds.: find the lengths of its sides.

(15) The diagonals of a rhombus are 8 inches and 5 inches respectively: find the lengths of its sides.

(16) An equilateral triangle has sides 5 inches long: find (i) the length of the perpendicular from a corner to the opposite side; (ii) the area of the triangle.

(17) O is the centre of a circle; ON a perpendicular on a chord AB; P a point on AB produced:

(i) given $ON = 2$ inches, $PA = 5$ inches, $PB = 3$ inches, find
(α) the length of the tangent from P to the circle;
(β) the length of PO.

(ii) given $AB = 3$ inches, $OA = 4$ inches, $OP = 6$ inches, find
(α) the length of the ON;
(β) the length of the tangent from P to the circle.

(18) In a circle of radius 3 inches a rectangle is inscribed, two of whose sides are 5 inches long: find the length of the other two sides.

(19) In a circle of radius $2\frac{1}{2}$ inches a rectangle is inscribed, whose length is an inch more than its breadth: find the area of this rectangle.

(20) C is the centre of a circle and O a point outside it. OP is a tangent and OAB a secant. If $CP=4$ inches, $CO=7$ inches, and $OA=4$ inches, find OB.

(21) A triangle ABC has sides $a=112$, $b=50$, $c=77$. AN is drawn perpendicular to BC. By equating the expressions for AN^2 obtained from triangles ABN and ACN

(i) find BN, CN; (ii) find AN; (iii) find area of the triangle.

(22) Generalize question 21 using a, b, c for the lengths of the sides of the triangle.

Ex. 65. PROBLEMS.

[Numbers 1 to 46 lead to simple (including simultaneous) equations: number 47 to 92 lead to quadratic equations: the rest are miscellaneous.]

(1) A man goes a journey at 20 mi./hr. If he went at 16 mi./hr. he would arrive 4 hrs. later. How far did he go?

(2) Find a fraction such that if 1 be taken from both numerator and denominator it reduces to $\frac{1}{2}$, and if 1 be added to the numerator only it reduces to $\frac{2}{3}$.

(3) Find a fraction such that if $\frac{1}{2}$ be added to the numerator it is increased by $\frac{1}{56}$, but if 1 is subtracted from the denominator it becomes $\frac{1}{2}$.

(4) If the numerator of a fraction is decreased by 2 and the fraction then increased by $\frac{5}{6}$ the result is $1\frac{1}{3}$. If the denominator is decreased by 2 the fraction reduces to $\frac{3}{4}$. What is the fraction?

(5) I buy a dozen eggs, but as two turn out bad I pay at the rate of $1\frac{1}{2}$d. each. What is the price of eggs?

(6) What is the price of a dozen bottles of ginger beer when I find that one bottle in each dozen being empty I have to pay at the rate of 3d. a doz. more?

(7) The sum of two fractions is $\frac{5}{8}$, and four times the smaller exceeds twice the larger by $\frac{1}{4}$. What are the fractions?

(8) The denominator of a certain fraction is greater by 2 than the numerator, and if 1 be subtracted from the numerator the fraction reduces to $\frac{4}{5}$. What is the fraction?

(9) A colonel wishing to form his men into a solid square finds he has 55 men over. If he increases the side of the square by 1, he has forty men too few. How many men are there in the regiment?

(10) The receipts at a concert were £57. 7s. 6d., reserved seats 3s. 6d., unreserved 2s. each. The whole number of seats sold was 450; how many of these were reserved?

(11) A bag contains £8. 12s. made up of half-crowns and shillings. If 6 half-crowns are added, the number of half-crowns is three times the number of shillings How many are there of each?

(12) A boy walks to school at the rate of 11 yds. in 9 secs. and is a minute late. If he had walked at the rate of 22 yds. in 15 secs. he would have been $\frac{1}{2}$ min. too soon. How far is it to school?

(13) A boy walks to school at the rate of 10 yds. in 8 secs. and is just in time. If he had walked at the rate of 22 yds. in 15 secs. he would have been 1 min. 18 secs. too early. How far is it to school?

(14) The forewheel of a carriage makes 15 revolutions more than the back wheel in 180 yds. Find the circumferences if they are in the ratio 3 : 4.

(15) Find a number such that if $\frac{3}{8}$ of it be taken from 26 and $\frac{1}{5}$ of the remainder of it from the original number, the latter result is double of the former.

(16) In an election, if $\frac{1}{10}$ of those who voted for A had refrained from voting, B would have been returned by a majority of 128; while if $\frac{1}{5}$ of those who voted for B had transferred their votes to A the latter would have been elected by a majority of 535. Which candidate was elected and by what majority?

(17) The ages of two boys are in the ratio of 7 : 3. Five years hence their ages will be as 5 : 3. In how many years will the ratio be 4 : 3?

(18) A man can walk from A to B and back in a certain time at the rate of 4 mi./hr. If he walks at 3 mi./hr. there and 5 mi./hr. back he requires 12 mins. more for the double journey. What is the distance from A to B?

(19) A man travels a certain distance at a certain rate. If he went 3 mi./hr. slower, he would take $\frac{1}{3}$ of the time as long again. How fast did he go?

(20) A, B, C do a piece of work between them. B does $\frac{2}{5}$ of the work more than A and together they do four times as much as C does. How much does A do?

(21) A man is accustomed to walk from his house to a railway station in 18 mins. On one occasion he is delayed at starting, but by walking 1 mi./hr. faster than usual he gets to the station in 14 mins. Find the distance from the house to the station and his usual pace.

(22) Find a number such that whether divided into 2 or 3 equal parts the product of the parts shall be the same.

(23) A number is divided in the ratio of 2 : 1 and also 3 : 1 : 1. The product of the parts is the same in both cases. What is the number?

(24) The times taken by two pipes to fill a cistern are as 3 : 4. They take $8\frac{4}{7}$ mins. when on together. Find their times.

(25) The ratio of two numbers is 5 : 2, their H.C.F. is 2471. Find them.

(26) If the H.C.F. of two numbers is equal to their difference and also to one-fifth of their sum, show that their L.C.M. is 6 times their H.C.F.

(27) Two people start simultaneously from A and B to walk to B and A and home again. The first time they meet 5 miles from A, and the second time 7 miles from A How far is it from A to B?

(28) Two people start simultaneously from A and B to walk to B and A and home again, A to B being 7 miles. They meet the first time 3 miles from A. How far from A do they meet the second time?

(29) A man walks $10\frac{1}{6}$ miles on a road in 2 hrs. 35 mins. and returns in $2\frac{1}{2}$ hrs. His rates of walking up-hill, down-hill, and on the level are $3\frac{1}{2}$, $4\frac{2}{3}$, 4 mi./hr. How much was up and down-hill, the ratio of the two being 4 : 3?

(30) A man bought a certain number of cows and pigs for £352. He sold the cows at a profit of 20 per cent. and the pigs at a loss of 25 per cent. Altogether he gained £47. How many of each did he buy, if the cows cost £15 each and the pigs £4?

(31) A bought 17 cows and 34 sheep for £340. He sold the cows at a loss of 10 per cent. and the sheep for a gain of 20 per cent. and altogether realised £346. 16s. What did he pay for each?

(32) The population of one town was three times that of another. At the next census the joint population had increased 4 per cent. If each had increased at the other's rate the total increase would have been 6 per cent. Find the rates of increase of each town.

(33) Two sums of money are invested, one at 5 per cent. S.I., and the other at 3 per cent., and £31 is realised. If half the latter sum had been invested with former, and the other half as before, £33 would have been realised. What were the two sums?

(34) If £650 are put out at S.I. at one percentage, and £400 at another, £40 is realised. If the investments are reversed only £38. 15s. is realised. What are the two percentages?

(35) The cost of carpeting a certain area with carpet of a certain width at 7s. 6d. a yard is $7\frac{1}{2}$ guineas; but to paper the same area with the addition of 300 sq. ft. is £4. 1s. 6d., the paper being the same width as the carpet and costing 6d. a foot. What is the area of the carpet and width of the paper?

(36) The area of a certain square exceeds that of a smaller one by 469 sq. ft. The difference of their perimeters is 56 ft. Find the side of the smaller one.

(37) A and B together do a piece of work in $2\frac{8}{11}$ days. C works twice as fast as A, and if A were to work alone for $1\frac{1}{2}$ days, C would finish in $2\frac{1}{4}$ days. How long would A and B take to do it separately?

(38) A and B together do a piece of work in $3\frac{3}{7}$ days. If they worked together for a day, and the next day A went on alone, they could then finish it together in 2 days. How long does each take separately over the work?

(39) Two men start simultaneously from two towns, 44 miles apart, one walking, the other on a bicycle. They meet in 2 hrs. 12 mins. If the walker had gone in the opposite direction the other would have overtaken him in 4 hrs. Find the rates of each.

(40) A man rows 16 miles down stream and back in $10\frac{2}{3}$ hrs. He knows he can row a mile against stream in the same time that he can row 3 with it. How fast does the stream flow?

(41) A steamer passing up a tidal river goes on one day for $\frac{1}{2}$ hr. in slack water and 2 hrs. against stream, and covers 15 miles. Another day it goes for $2\frac{1}{2}$ hrs. with the stream and covers $37\frac{1}{2}$ miles. Find the river's rate, considered uniform, and the rate of the steamer in still water.

(42) A starts to catch up B who is 70 yards ahead. They stride in step but 8 of A's strides are equal to 11 of B's. If A catches B in 140 strides how far does each go at a stride?

(43) Two crews A and B row a race. A pulls 7 strokes while B pulls 6, but 7 of A's strokes equal 8 of B's. If A wins by 42 yards while B pulls 36 strokes, find the distance each crew goes at a stroke.

(44) A dog starts in pursuit of a hare at a distance of 30 of his own leaps from her. He take 5 leaps while she takes 6 but covers as much ground in 2 as she in 3. In how many leaps of each will the hare be caught?

(45) Two vessels contain mixtures of whiskey and water. In one there is three times as much whiskey as water and in the other four times as much water as whiskey. How much must be taken from each to fill a half-pint flask so that the contents may be half whiskey and half water?

(46) Two vessels each contain wine and water, one in the ratio of 2 : 3 and the other in the ratio of 2 : 7. What quantity must be taken from each so that the mixture shall contain 6 gallons of wine and 13 of water?

(47) Find four consecutive numbers such that the product of first and last is less by 19 than the number which has the two middle ones for its digits.

(48) How can you form 730 men into two squares so that the front of one presents four more men than the front of the other?

(49) I buy a number of pencils for 10s. If I had bought half a dozen more for the same money they would have cost $\frac{1}{30}$d. less each. How many did I buy?

(50) I buy a number of Fives balls for £2. 2s. I should have got two dozen more for the money if they had cost $\frac{1}{2}$d. less each. What did they cost each?

(51) If three more eggs in a shilling's worth lowers the price 4d. per dozen, what is the cost of a dozen eggs?

(52) How many eggs do I get for 1s., if finding three bad I pay at the rate of $\frac{1}{2}$d. more per egg?

(53) I bought a number of eggs for 5s. If I had got a dozen less for my money I should have paid at the rate of $1\frac{3}{7}$d. more per dozen. How many did I buy?

(54) A man bought a number of cricket bats for £62. 10s., and sold them again for 15s. each. He gained the price of 20 bats. How much did he pay for each?

(55) A man spends £52. 10s. on coals, and finds that when the price rises 14s. per ton he gets 20 tons less than before. What is the price per ton?

(56) A man travels 116 miles at a certain rate; but if he had gone 2 mi./hr. faster he would have arrived 2 hrs. sooner all but 4 mins. How fast did he go?

(57) A man has to walk 10 miles. After the first mile he increases his speed $\frac{1}{2}$ mi./hr., and is therefore at his destination $\frac{1}{4}$ hr. sooner. How long did he take?

(58) One side of a rectangle is 9 ins. shorter than the diagonal and 9 ins. longer than the other side. Find the area.

(59) The diagonal of a rectangular field is 27 yds. longer than the longer side and 54 yards longer than the shorter side. Find the area of the field.

(60) One side of a rectangle is 9 ins. shorter than the diagonal and 5 ft. 3 ins. longer than the other side. Find the area of the rectangle.

(61) The length of a wooden brick is twice its breadth. If 10,560 such bricks are required to pave a hall 99 ft. by 30 ft., what is the length of a brick?

(62) A hall has to be paved with square tiles of which there are two kinds, one 8 ins. square and the other larger. If the larger ones are used then 192 will be required; if the smaller ones then 243. What is the size of the larger ones?

(63) A room is 6 ft. longer than broad, and has a square carpet in the middle whose side is 12 ft. The rest is decorated with square blocks of wood, the side of each being 6 ins. What are the dimensions of the room if 1152 such blocks are required?

(64) A hall 30 ft. by 20 ft. is paved with wooden blocks four times as long as broad. If the breadth remained the same and the length was five times the breadth, the number of blocks required would be 270 less. Find the dimensions of the blocks originally.

(65) A parquet floor 24 ft. by 18 ft. is made of blocks of wood 12 ins. longer than broad. If it takes 972 such blocks, how large are they, and how large ought *square* blocks to be cut so that the same number might be used?

(66) Part of a garden is rectangular and composed of a gravel path 6 ft. wide running all round, and a lawn in the middle. A marquee is placed in the middle of the lawn so that there is a border of grass all round 12 ft. deep. What is the distance all round the extremity of the gravel path if the marquee is 20 yds. longer than wide, and contains an area of 476 sq. yds.?

(67) A swimming bath is 44 ft. longer than it is broad. There is a distance of 9 ft. from the edge to the dressing rooms, which are all round the bath, and are themselves 9 ft. deep. The whole floor surface has an area of 928 sq. yds. What are the dimensions of the swimming bath itself?

(68) A hall is 25 ft. by 40 ft., and has a parquet floor composed of squares, *each* square having a border of dark wood all round of a certain depth, and a light square in the middle. A square with its border has a side of 15 in. If the hall were paved with the light squares only, the number of squares would be 360 more than they are. How deep is the border?

(69) The area of a certain square together with that of a smaller one is 1898 sq. ft. The difference in perimeter is 56 ft. Find the side of the larger one.

(70) The times which two pipes take to fill a cistern differ by 2 mins. They take $4\frac{4}{9}$ mins. to fill it if turned on together. Find the time taken by each.

(71) What is the size of a cubical cistern such that if it were made 3 ft. longer, 2 ft. narrower, and the same depth, it would contain the same amount of water?

(72) The breadth of a large block of ice is twice the depth, and the length twice the breadth. If it were cut into smaller blocks $1\frac{1}{2}$ ft. $\times$ 1 ft. $\times$ 9 in. there would be 576 less blocks than if the dimensions were 9 ins. $\times$ 9 ins. $\times$ 6 ins. How big is the original block?

(73) The large wheel of a carriage is $1\frac{1}{2}$ ft. more in circumference than the small one, and makes $62\frac{8}{7}$ fewer revolutions in a mile. Find the circumference of each wheel.

(74) The wheel of a high bicycle is 16 ft. in circumference. If it took $\frac{1}{11}$ secs. less to revolve the rider would travel $\frac{10}{11}$ mi./hr. faster. At what rate is he going?

(75) A man walking in a hill country comes to a sign post at P, where the distances are marked in hours instead of miles—but he mistakes the reading for *miles*. He is walking at the average rate assumed on the sign post, but owing to his mistake the journey takes him $1\frac{1}{2}$ hrs. longer than he expected. The distance is really eight miles. What was the reading on the sign post?

(76) In the preceding question if his mistake made him 1 hr. 20 mins. later than he expected, and the average rate of walking was 3 mi./hr., how far was it from P to his destination?

(77) A steamer leaves C for D (110 miles) at the same time as one leaves D for C. They meet at a port P, and get to their respective destinations, the one 6 hrs. afterwards, and the other 4 hrs. 10 mins. How fast are they travelling?

(78) The distance between two towns, A and B, is 420 miles. A motor car leaves A for B at the same time that a bicycle leaves B for A. The motor car arrives at its destination 9 hrs. after the two vehicles meet, and the bicycle 16 hrs. after. What is the rate of each?

(79) When two trains meet, their engines pass one another at 75 mi./hr. One train has 90 miles to go to its destination, and increases its speed by 5 miles per hour; the other, which has only 45 miles to go, keeps the same rate and gets to its destination 18 mins. before the other. Find their original rates.

(80) A and B sell 100 sheep. If A sold his per head for as many shillings as B bought sheep and *vice versa*, they would have together realised £240. How many sheep did each sell?

(81) Find a number of two digits which equals 9 times the sum of the digits, and if diminished by 32 equals the square of the difference of the digits.

(82) Find a number of two digits which equals 7 times the sum of the digits, and if increased by 18 equals the square of the sum of the digits.

(83) If 1 is added to the denominator of a fraction it is diminished by $\frac{3}{28}$. If 1 is added to the numerator it becomes unity. Find the fraction.

(84) The sum of the digits of a number less than 100 is 17, and the square of the ten's digit is less than the square of the unit's digit by 17 also. What is the number?

(85) The difference between the squares of the digits of a number is equal to their sum, and the number itself is five times the sum of the digits. What is the number?

(86) The sum of two numbers is 32, and the sum of their squares is 514. Find the numbers.

(87) The difference between two numbers is 9, and the difference of their squares is 207. What are the numbers?

(88) In a certain election A's poll equals the square of his majority over B, and there are altogether 10,935 electors. What is A's poll and majority?

(89) After passing at P, A and B are due to arrive simultaneously at their destinations, which are distant from P 27, 24 miles respectively: but B reduces his pace by $\frac{1}{2}$ mi./hr. and A by 1 mi./hr., and consequently B arrives $\frac{6}{7}$ hour before A. How fast was each walking originally?

(90) A hall is 36 ft. by 15 ft. It is paved with square tiles, half of it with one size and half with another. There are 210 more of the smaller used than of the larger. Twelve more of the smaller would go along the length of the hall than the larger. What is the length of a side of each?

(91) A number of oranges was bought for 2s. $7\frac{1}{2}$d.; half of them cost 3d. a dozen more than the other half, and four more of the cheaper were got for 1s. than of the dearer. How many oranges were bought?

(92) The perimeter of a rectangular field is 434 yds. If the diagonal is 155 yds., what is the area?

(93) Find four consecutive numbers such that the product of the last two is a number which has the first two for its digits.

(94) A man having to walk 10 miles finds that he can arrive $16\frac{2}{3}$ mins. sooner than he otherwise would by increasing his speed $\frac{1}{2}$ mi./hr. What time will he take if he only begins to quicken his pace half way?

(95) A ladder 34 ft. long just reaches a window when placed in such a position that the height of the window above the ground exceeds the distance of the foot of the ladder from the wall by 14 ft. Find the height of the window.

(96) A number of soldiers when formed into a solid square present 16 men fewer in front than when formed into a hollow square four deep. How many men are there?

(97) A man rows $3\frac{1}{2}$ miles down a river and back in 1 hr. 40 mins. The stream runs at 2 mi./hr. What is his rate in still water?

(98) Two men row a time race over a course of $1\frac{1}{4}$ miles. Their speeds are as 45 : 44, and A wins by 10 secs. Find the speed of each in yards per second.

(99) The incomes of A and B are as 3 : 2, and their expenditure as 5 : 3. Each saves £1000 a year. Find their incomes.

(100) At a review of an army the troops were in a solid mass 40 deep, and there were just a quarter as many men in front as there were spectators. If the spectators had been drawn up with the army and the depth increased by 5 the number of men in front would have been 100 fewer than before. Find the number in the army.

(101) Two gallons of brandy are drawn from a cask, and it is filled up with water. Two gallons of the mixture are then taken and it is again filled up with water. If the proportion of brandy to water in the end is as 16 : 9, how much brandy was there originally?

(102) A gentleman has 20 teaspoonfuls of whiskey in a flask. He takes out a certain amount and fills up with water. Of the mixture he again takes the same amount as before, and the proportion of water to whiskey is 361 : 39. How much did he take out at first?

(103) A and B buy cloth. A pays 41s. more for 60 yds. than B does for 28, and A gets 4 yds. more than B for 2 guineas. How much does each pay per yard?

(104) A labourer saves 5 per cent. of his wages. After a strike his wages are raised 1s. a week, but the cost of living being increased in the ratio of 20 : 19 he finds that he saves 26s. a year less than before. What were his weekly wages?

(105) There are two pieces of metal each containing a mixture of gold and silver. The first contains 6 ozs. of gold and 10 ozs. silver, and the second 15 ozs. of gold and 17 ozs. of silver. How much must be taken from each to form a compound of 5 ozs. of gold and 6 ozs. of silver?

(106) In working a simple equation the answer obtained is 8; but on looking through the work a mistake is found in the part not involving x, and when this is corrected the answer obtained is 6. A second mistake is then discovered in the coefficient of x, and if this alone were corrected the answer would be 2. What is the right answer?

(107) In a constituency in which each elector may vote for two candidates, $\frac{1}{3}$ of the electors vote for A, and divide their votes among B, C, D, E in proportion 4, 3, 2, 1. Of the remainder, $\frac{2}{3}$ vote for B and divide their votes between C, D, E, in proportion 3, 1, 1. Half of the remainder vote for D and E, and 540 do not vote at all. Find the whole number of electors and the order of the poll.

(108) A and B run a 220 yds. race. After they have been going for $13\frac{4}{9}$ secs., A trips up, and while he recovers himself B gains $2\frac{4}{9}$ yds. When A restarts (at the same pace as before) B is going a yard a second faster than A. They continue these rates and come in a dead heat. How many yards a second was each going at the start, if the ratio of their rates was 9 : 8?

(109) A man buys 60 tons of coal and 71 of coke for £117. 12s. For £15 he gets 13 more tons of coke than coal. Find the price of each per ton.

Ex. 66. SURDS.

[It is suggested that in these examples the use of square-root tables be permitted.]

(1) Express with the smallest possible numbers under the $\surd$ sign:

(i) $\sqrt{8}$; (ii) $\sqrt{162}$; (iii) $\sqrt{27}$; (iv) $\sqrt[3]{32}$; (v) $\sqrt[3]{81}$;

(vi) $\sqrt{320}$; (vii) $\sqrt[3]{686}$; (viii) $\sqrt[5]{64}$; (ix) $3\sqrt{200}$; (x) $5\sqrt{48}$.

(2) Find the numerical values of the first three expressions in question (i), first, by using the original, and, secondly, by using the final form of the expressions.

(3) Find the numerical values of

(i) $\sqrt{2}+2\sqrt{5}-\sqrt{7}$; (ii) $6\sqrt{10}-8\sqrt{9}+7\sqrt{8}$;

(iii) $4\sqrt{2}-6\sqrt{3}+3$; (iv) $\sqrt{36}-5\sqrt{100}+40\sqrt{2}$.

(4) Simplify and find the numerical values of

(i) $\sqrt{2}\times\sqrt{3}$; (ii) $\sqrt{7}\times\sqrt{5}$; (iii) $\sqrt{8}\times\sqrt{32}$;

(iv) $\sqrt{7}\times\sqrt{21}$; (v) $\sqrt[3]{3}\times\sqrt[3]{9}$; (vi) $\sqrt[3]{49}\times\sqrt[3]{7}$;

(vii) $\sqrt{8}-\sqrt{2}$; (viii) $3\sqrt{27}-7\sqrt{3}$; (ix) $\dfrac{22}{\sqrt{2}}$;

(x) $\dfrac{42}{\sqrt{6}}$; (xi) $\dfrac{9}{\sqrt{5}}$; (xii) $\dfrac{1}{\sqrt{200}}$;

(xiii) $\dfrac{\sqrt{3}}{\sqrt{2}}$; (xiv) $\dfrac{1}{5\sqrt{2}}$; (xv) $\dfrac{64\sqrt{2}}{\sqrt{128}}$.

(5) Simplify

(i) $\sqrt{a}\,.\sqrt{a^3}$; (ii) $(x\sqrt{a}\,.\,a\sqrt{x})^2$; (iii) $\sqrt{a^3b}\div\sqrt{ab^3}$;

(iv) $a\sqrt{bc}\,.\,b\sqrt{ac}\,.\,c\sqrt{ab}$; (v) $a\sqrt{x^3a}\,.\,b\sqrt{x^3b}$; (vi) $\left(\dfrac{a\sqrt{x}}{x\sqrt{a}}\right)^3$;

(vii) $\sqrt{a^2bc}\,.\,\sqrt{ab^2c}\,.\,\sqrt{abc^2} \div \sqrt{abc}$; (viii) $\sqrt{y^2z}\,.\,\sqrt{z^2x}\,.\,\sqrt{x^2y} \div \sqrt{xyz}$;

(ix) $\sqrt[3]{a^3b^2c}\,.\,\sqrt[3]{a^2b^3c} \div \sqrt[3]{a^2b^2c^2}$; (x) $\sqrt[3]{x^4y}\,.\,\sqrt[3]{xy^4} \div \sqrt[3]{xy^2}$.

Ex. 67.

(1) Simplify and find the numerical value of

(i) $(\sqrt{7}-\sqrt{5})\sqrt{7}$; (ii) $(1+\sqrt{2})\sqrt{2}$; (iii) $(3\sqrt{2}+1)\sqrt{2}$;

(iv) $(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})$; (v) $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$;

(vi) $(2\sqrt{3}-1)(2\sqrt{3}+1)$; (vii) $(\sqrt{3}+\sqrt{2})^2$;

(viii) $(5\sqrt{3}-\sqrt{7})(5\sqrt{3}+\sqrt{7})$; (ix) $(2\sqrt{2}-1)^2$.

(2) Simplify

(i) $(a\sqrt{x}+\sqrt{x^3})\sqrt{x}$; (ii) $(\sqrt{x}+\sqrt{y})\sqrt{x}$;

(iii) $(\sqrt{x}+\sqrt{a})(\sqrt{x}-\sqrt{a})$; (iv) $(b\sqrt{a}+a\sqrt{b})(b\sqrt{a}-a\sqrt{b})$;

(v) $(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})$; (vi) $(\sqrt{x}+\sqrt{y})^2$;

(vii) $(\sqrt{x+y}+\sqrt{x-y})^2$; (viii) $(\sqrt{a^2-1}-\sqrt{a^2+1})^2$;

(ix) $\{a\sqrt{x-y}+b\sqrt{x+y}\}\{a\sqrt{x-y}-b\sqrt{x+y}\}$;

(x) $\{a\sqrt{x+y}-b\sqrt{x-y}\}\{b\sqrt{x+y}+a\sqrt{x-y}\}$.

(3) Rationalize the denominator and find the numerical value of

(i) $\dfrac{1}{\sqrt{5}-\sqrt{3}}$; (ii) $\dfrac{1}{\sqrt{3}+1}$; (iii) $\dfrac{2}{\sqrt{7}-\sqrt{5}}$;

(iv) $\dfrac{2}{1+\sqrt{3}}$; (v) $\dfrac{5}{\sqrt{5}+1}$; (vi) $\dfrac{\sqrt{7}+\sqrt{5}}{\sqrt{7}-\sqrt{5}}$;

(vii) $\dfrac{1+\sqrt{10}}{1-\sqrt{10}}$; (viii) $(2+\sqrt{2})\,.\,\dfrac{2\sqrt{2}-1}{4\sqrt{2}+1}$.

(4) Rationalize the denominators of

(i) $\dfrac{1}{a+\sqrt{x}}$; (ii) $\dfrac{\sqrt{a}}{\sqrt{a}+\sqrt{b}}$; (iii) $\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$;

(iv) $\dfrac{a+b\sqrt{x}}{x-b\sqrt{a}}$; (v) $\dfrac{1}{\sqrt{a+b}-\sqrt{a-b}}$; (vi) $\dfrac{\sqrt{1+a^2}-\sqrt{1-a^2}}{\sqrt{1+a^2}+\sqrt{1-a^2}}$.

(vii) $\dfrac{2x^2}{\sqrt{x^2+y^2}+\sqrt{x^2-y^2}}$; (viii) $\dfrac{1}{a+\sqrt{b}+\sqrt{c}}$.

(5) Write down in the form $\sqrt{a}+\sqrt{b}$ the square roots of

(i) $12+2\sqrt{35}$; (ii) $5-2\sqrt{6}$; (iii) $9+2\sqrt{18}$; (iv) $9+6\sqrt{2}$;

(v) $11+6\sqrt{2}$; (vi) $8-2\sqrt{7}$; (vii) $9+2\sqrt{14}$; (viii) $23-2\sqrt{130}$.

Find also the numerical values in numbers (iii) and (v), (α) by using the original form of the expression, and (β) by using the form $\sqrt{a}+\sqrt{b}$.

Ex. 68. INDICES.

Oral. (Revision of laws of indices.)

(1) What are:

(i) $a^8 . a^3$; (ii) $b^{11} \div b^4$; (iii) $(xy)^3 \div x$;

(iv) $c^7 . d^8 \div (cd)^2$; (v) $\left(\frac{x}{y}\right)^8$; (vi) $x^9y^7 \div (xy^2)^3$.

(2) How many zeros have (i) $10^8 . 10^9$; (ii) $10^{21} \div 10^{17}$; (iii) $(10^2)^7$?

(3) What is the square of

(i) 10^{17}; (ii) 3^6; (iii) 2^7; (iv) 14^2; (v) a^6; (vi) xy^2?

(4) What is the square root of

(i) 10^4; (ii) 8^{32}; (iii) 6^4; (iv) 13^{12}; (v) a^{14}; (vi) b^{2x}; (vii) c^{8x}?

(5) What is the cube root of

(i) 10^6; (ii) 2^{24}; (iii) a^9; (iv) b^{3y}; (v) c^{12x}?

(6) Express as powers of 2

(i) 4^3; (ii) 8^3; (iii) 16^2; (iv) 32^4?

(7) What powers of 8 are (i) 2^{24}; (ii) 4^{15}; (iii) 16^6?

(8) Between what powers of 27 do the following numbers lie:

(i) 3^7; (ii) 3^{44}; (iii) 3^{71}?

(9) Between what powers of 10 do the following numbers lie

(i) 2630; (ii) 40; (iii) 6; (iv) 460,008; (v) 53·25?

(10) Between what powers of 8 do the following numbers lie

(i) 2^7; (ii) 2^{17}; (iii) 2^{24}; (iv) 2^{31}?

Ex. 69. FRACTIONAL INDICES.

Oral.

(1) What are: (i) $4^{\frac{1}{2}}$; (ii) $25^{\frac{1}{2}}$;
(iii) $32^{\frac{1}{5}}$; (iv) $27^{\frac{1}{3}}$; (v) 9^{-2}; (vi) 6^{-1};
(vii) 4^{0}; (viii) $64^{-\frac{1}{2}}$; (ix) $8^{-\frac{1}{3}}$; (x) $16^{\frac{1}{4}}$?

(2) What are: (i) $8^{\frac{2}{3}}$; (ii) $27^{-\frac{2}{3}}$;
(iii) $64^{-\frac{5}{6}}$; (iv) $16^{-\frac{3}{4}}$; (v) $100^{\frac{3}{2}}$; (vi) $16^{\frac{5}{4}}$;
(vii) $4^{2\frac{1}{2}}$; (viii) $16^{1\cdot5}$; (ix) $32^{1\cdot2}$; (x) $16^{1\cdot25}$?

(3) Express in their simplest forms, with positive indices:
(i) $(a^9x^{15})^{\frac{1}{3}}$; (ii) $(a^7)^{-2}$; (iii) $(x^8)^{-\frac{3}{2}}$; (iv) $(16a^{12})^{\frac{3}{4}}$;
(v) $(8x^9)^{-\frac{2}{3}}$; (vi) $(8x^{-9})^{\frac{2}{3}}$; (vii) $(8x^{-9})^{-\frac{2}{3}}$; (viii) $(9x^8)^{\frac{3}{2}}$;
(ix) $(9x^{-8})^{-\frac{3}{2}}$; (x) $(9x^{-8})^{\frac{3}{2}}$.

(4) Given $\sqrt{7}$ (as 2·646...), explain how to find
(i) $28^{\frac{1}{2}}$; (ii) $63^{\frac{3}{2}}$; (iii) $14^{\frac{1}{2}}.18^{\frac{1}{2}}$.

(5) $\sqrt{7}$, $\sqrt{3}$, and $\sqrt{2}$ being supposed given, explain how to find
(i) $12^{\frac{1}{2}}$; (ii) $63^{\frac{3}{2}}$; (iii) 8^{-3}; (iv) $84^{-\frac{1}{2}}$; (v) $324^{\frac{1}{4}}$.

Ex. 70.

(1) Find the values of the following to three places of decimals (taking square root when necessary):
(i) $8^{\frac{1}{2}}$; (ii) $12^{\frac{1}{2}}.3^{2\frac{1}{2}}$; (iii) $225^{\frac{1}{2}}$;
(iv) $32^{-\frac{1}{2}}$; (v) $33^{\frac{1}{2}}.132^{\frac{1}{2}}$; (vi) $35^{\frac{1}{2}} \div 7^{-\frac{3}{2}}$.

(2) Find by repeatedly taking square root the values of $10^{\frac{1}{2}}$, $10^{\frac{1}{4}}$, $10^{\frac{1}{8}}$, $10^{\frac{1}{16}}$ to four places of decimals. [For safety each square root should be found to one more place than is required for the next: thus $10^{\frac{1}{2}}$ should be found to seven decimal places.]

(3) Verify by finding (from the definition) the numerical value of each side (using square-root tables):
(i) $10^{\frac{1}{2}}.10 = \sqrt{10^3}$; (ii) $8^{\frac{1}{2}} = 2\sqrt{2}$;
(iii) $10^{\frac{1}{2}} = 10 \div 10^{\frac{1}{2}}$; (iv) $10^{\frac{1}{2}}.10^{\frac{1}{4}} = \sqrt[4]{10^3}$.

(4) Tabulate the numerical values of $10^{\frac{1}{5}}$, $10^{\frac{2}{5}}$, $10^{\frac{3}{5}}$,... up to $10^{\frac{7}{5}}$ to three places of decimals.

Hence draw the graph of $y = 10^x$ on a large scale for values of x between 0 and 1 (*e.g.* take unit for y 1 inch, unit for x 10 inches).

(5) Use the graph of question 4 to find the value of

(i) $10^{\cdot 2}$; (ii) $10^{\cdot 4}$; (iii) $10^{\cdot 6}$; (iv) $10^{\cdot 8}$; (v) $10^{\cdot 23}$.

(6) Use the graph of question 4 to express as powers of 10 the following numbers:

(i) 2; (ii) 3; (iii) 5; (iv) 7; (v) 4·26; (vi) 1·74; (vii) 5·83.

(7) Show how from the results of question 6 the following numbers can be expressed as powers of 10:

(i) 20; (ii) 3000; (iii) ·5; (iv) ·07;
(v) 4260; (vi) 174; (vii) ·00583.

(8) Use the 4-figure logarithm table to check the results of questions 5 and 6.

(9) Find from the tables the values of

(i) $10^{\frac{1}{2}}$; (ii) $10^{2\cdot 62}$; (iii) $10^{\frac{1}{5}}$; (iv) $10^{-1\cdot 213}$; (v) $10^{1\cdot 08}$.

[Further practice with fractional indices may be obtained by working through Ex. 73, using *index notation* throughout.]

Ex. 71. LOGARITHMS.

(1) Arrange as

(number between 1 and 10) × (integral power of 10)

the following numbers: 1364, 300, 27 million, ·0000364, 8001, ·0267, ·99, 2837·12, sixteen and a half million, $\cdot 0\dot{3}$, $\cdot 1\dot{2}\dot{7}$.

Make a table such as the following:

Number.	Same in standard form.	Logarithms.
47·8	$4\cdot 78 \times 10^1$	1·6794
3625	$3\cdot 625 \times 10^3$	3·5593
2417356	$2\cdot 417 \times 10^6$	6·3832
·0036519	$3\cdot 652 \times 10^{-3}$	$\bar{3}\cdot 5625$

showing the standard forms and the logarithms of the following numbers. Check by using the antilogarithm table:

(2) 26·4, 204, 5·68, 53600.

(3) ·03, ·00021, ·00647, ·239.

(4) 18 65, 23620, 1421000, 32·4312.

(5) ·013, ·26412, 1·93612, $\dot{3}$.

(6) ·000004, 184265, $\dot{7}$, ·08628.

Ex. 72.

Use logarithms to multiply the numbers 1, 2, 3, 4, 5 by the numbers *a*, *b*, *c*, *d*, *e*; check by ordinary multiplication:

(1)	2·645,	(*a*)	2,
(2)	7·98,	(*b*)	3,
(3)	462·7,	(*c*)	·08,
(4)	5360000,	(*d*)	700,
(5)	·00371.	(*e*)	$\dot{5}$.

Use logarithms to divide the numbers 6, 7, 8, 9, 10 by the numbers *a*, *b*, *c*, *d*, *e*; check by ordinary division:

(6)	3·86,	(*a*)	5,
(7)	43·92,	(*b*)	2,
(8)	·06135,	(*c*)	6000,
(9)	·317,	(*d*)	·03,
(10)	630400.	(*e*)	·0001.

Use logarithms to find the squares of the following numbers; check by using the table of squares:

(11) 7·821, 642·9, ·0136, ·0007241, 83000.

(12) 42·7, 8493000, ·187, ·0936, ·0000542.

Use logarithms to find the square roots of the following numbers; check by using the table of square roots:

(13) 579·2, 869000, ·9321, ·07929, 64·39.

(14) 46·35, ·0836, ·836, 97240, 97·24.

Ex. 73.

On each of the numbers 1, 2, 3,... 7 perform the operations indicated by the signs before the numbers a, b, c, .. g.

(1)	273·2,	(a)	$\times$ 63·87,
(2)	938000,	(b)	$\div$ 134·6,
(3)	·0007163,	(c)	$\div$ ·0042,
(4)	8·42073,	(d)	$\times$ 1·6423,
(5)	31·6,	(e)	$\times$ 7480000,
(6)	·5007,	(f)	$\times$ ·263,
(7)	·01832.	(g)	$\div$ 09341.

(8) The inventor of the game of chess is said to have been granted as a reward by the oriental monarch whom he served, that he should be given 1 grain of corn for the 1st square of the chess-board, 2 grains for the 2nd, 4 grains for the 3rd, 8 grains for the 4th, and so on to the last (64th) square.

Find the total number of grains of corn that he should have received [the number is almost exactly double the number to be received for the last square: to 7 figures $\log 2 = \cdot 3010300$].

(9) Find the amount of £2300 at 5 % compound interest at the end of 5, 10, 15, 20, and 25 years respectively.

(10) Find the true present worth of £1800 due 10 years hence, reckoning compound interest at 3 %.

(11) For the rates % (i) 3, (ii) 5 find the times required for a sum of money to (α) double itself, (β) treble itself, compound interest being reckoned.

Ex. 74. DEGREE, Etc.

(1) Find the terms of the highest degree in the product

$$(1+x-y+xy)(1+x+y)(2x+2y+x^2+y^2);$$

also those of the lowest degree.

(2) Find the sum of the coefficients in each of the following products:

(i) $(a+2b)(3a+b)(b+a)^3$;

(ii) $(2x+y)(x+2y)(x^2+3xy+y^2)$;

(iii) $(a+b+c)(a^2+b^2+c^2-bc-ca-ab)$;

(iv) $(x+y+z)^2(x^2+y^2+z^2-2yz-2zx-2xy)$.

(3) If Σ refers to the four letters x, y, z, w, write down in full Σx^2, Σyz, Σxyz; and show that $(\Sigma x)^2 = \Sigma x^2 + 2\Sigma yz$.

(4) If Σ refer to the cyclic interchanges of the letters a, b, c, write in full $\Sigma(a^2-bc)$, $\Sigma(a^3-b^2c)$, Σa^3b^2c.

(5) Show by rearranging that each of the following has $b-c$ as a factor:

(i) $a^2(b-c)+b^2(c-a)+c^2(a-b)$;

(ii) $a^3(b-c)+b^3(c-a)+c^3(a-b)$;

(iii) $a^4(b-c)+b^4(c-a)+c^4(a-b)$.

What can be at once inferred as to the other factors?

(6) What condition is necessary that $(ax+by+cz)^2$ should be symmetric with respect to y and z?

(7) Find the value of

$$(a^2+b^2)(x^2-y^2)+(b^2+c^2)(y^2-z^2)+(c^2+a^2)(z^2-x^2).$$

(8) Show that

$$(a+b+c)(x+y+z)-(ax+by+cz) = a(y+z)+\ldots$$
$$= x(b+c)+\ldots.$$

(9) Simplify

$$(a^2-b^2)(2z+x+y)+(b^2-c^2)(2x+y+z)+(c^2-a^2)(2y+z+x).$$

(10) If Σ refers to the three letters x, y, z, find the value of

$$(\Sigma x)(\Sigma x^2)-\Sigma x^3-2(x+y)(y+z)(z+x).$$

Ex. 75. USE OF THEORY OF FORM AS CHECK.

Point out obvious mistakes in the following untrue statements

(1) $(x^2+y^2)(x+y)(xy) = x^4y-x^3y^2+x^2y^3-xy^4$.

(2) $(a^2+b^2)(2ab-a^2-b^2) = 2a^3b+2ab^3-a^3-b^3-2a^2b^2$.

(3) $(a-b)(b-c)(c-a) = 2abc-a^3-b^3-c^3$.

(4) $(a+b+c)(a^2+b^2+c^2-2abc) = a^3+b^3+c^3-3abc$.

(5) $(x+y+z)^2(yz+zx+xy) = xyz\Sigma x+2\Sigma y^2z^2+\Sigma z\Sigma yz$.

(6) $(a^2+b^2+c^2)(a+b+c) = a^3+b^3+a^2(b+c)+b^2(c+a)+c^2(a+b)$.

Point out obvious mistakes in the following untrue statements.

(7) $(ax+by+cz)(a+b+c)=a^2x+b^2y+c^2z+bcy+ca(z+x)+abx.$

(8) $(x+y+z)^2(ax+by+cz)=\Sigma ax^3+\Sigma a^2x^2+2xyz\Sigma a.$

(9) $(a-b)(b-c)(c-a)=(a+b+c)^3-(a^2+b^2+c^2)(a+b+c).$

(10) $(x^n+y^n)(x^n+z^n)(x^n+w^n)=(x^n+y^n+z^n+w^n)x^{2n}+x^n(y^n+w^n).$

Ex. 76. INDETERMINATE COEFFICIENTS.

(1) If $n^3=An(n-1)(n-2)+Bn(n-1)+Cn$ for all values of n: find A, B, C.

(2) Express $2x^3+3x^2-6x+4$ in the form

$$a+b(x+1)+c(x+1)^2+d(x+1)^3,$$

where a, b, c, d do not contain x.

(3) If $(x+y-z)(y+z-x)(z+x-y)(x+y+z)$

$$\equiv A(x^4+y^4+z^4)+B(y^2z^2+z^2x^2+x^2y^2),$$

find A and B.

(4) If $(a+b+c)(a^2+b^2+c^2-bc-ca-ab)$

$$=A\Sigma a^3+B\Sigma a^2b+C\,.\,abc,$$

find A, B, C.

(5) Express n^4-n^2 in the form

$$A+Bn+Cn(n+1)+Dn(n+1)(n+2)+En(n+1)(n+2)(n+3).$$

(6) Find l, m, n so that x^2+3x+2 may be the same as

$$lx(x-1)+mx(x+1)+n(x-1).$$

(7) Find a, b, c, d so that

$$x^2-3x+4\equiv a+b(x-1)+c(x-1)^2+d(x-1)^3$$

(8) If $(a+b)^3\equiv Aa^3+Ba^2b+Cab^2+Db^3,$

why do we know at once that $A=D$ and $B=C$? Show how to find A, B without multiplying out.

(9) If $(a+b+c)^3=A\Sigma a^3+B\Sigma a^2b+C\,.\,abc,$

find A, B by putting $c=0$ and find C.

(10) Use undetermined coefficients to find the expansion of $(a+b+c+d)^3$.

Ex. 77. THE FUNCTIONAL NOTATION.

(1) If $f(x)=x^3-3x$, write down and find value of $f(1)$, $f(2)$, $f(3)$.

(2) If $f(n)=(n-1)(n+1)+3n^2$, write down and find value of
$$f(-1),\ f(0),\ f(7).$$

(3) If $f(x)=ax^2+bx+c$, write down
$$f(2),\ f(y),\ f(x+1).$$

(4) If $\phi(n)=n^2+n+1$, write down and simplify
$$\phi(n+1) \text{ and } \phi(n-1),$$
and show that $\phi(n+1)+\phi(n-1)=2\phi(n)+2.$

(5) If $\phi(x)=ax^2+2a^2x+3a^3$, write down and simplify
$$\phi(-a),\ \phi(0),\ \phi(a),\ \phi(2a).$$

(6) If $f(x)=3x^2+4x+2$, show that
$$f(x+1)+f(x-1)=2f(x)+6.$$

(7) If $f(n)=an^2+bn+c$, show that
$$f(n+1)+f(n-1)=2f(n)+2a.$$

(8) If $\phi(n)=an^3$, find the value of
$$\phi(n+1)+\phi(n-1)-2\phi(n).$$

(9) If for all values of n, s_n stand for x^n+y^n and p_n for x^ny^n, prove that $s_n \cdot s_m = s_{n+m} + p_m \cdot s_{n-m}$,
and that $s_1(s_{10} \cdot s_8 - s_9^2) = s_{11} \cdot s_8 - s_{10} \cdot s_9$.

Ex. 78. REMAINDER THEOREM.

Find the remainders when the functions (1), (2), (3), (4), (5) are divided by the binomials (*a*), (*b*), (*c*), (*d*), (*e*) respectively, and similarly in the other groups.

(1)	x^2-x+1,	(*a*)	$x-1$,
(2)	$2x^3+2x^2-3x-1$,	(*b*)	$x-3$,
(3)	x^4-1,	(*c*)	$x-2$,
(4)	x^3-7x-6,	(*d*)	$x+1$,
(5)	x^3+x^2-4x-4.	(*e*)	$x+2$.
(6)	$x^2+2ax+a^2$,	(*a*)	$x-a$,
(7)	$x^3+2ax^2-9a^2x-18a^3$,	(*b*)	$x+a$,
(8)	x^9-a^9.	(*c*)	$x-3a$.
(9)	$x^3+3xy^2+3x^2y+y^3$,	(*a*)	$x-2y$,
(10)	x^6-y^6,	(*b*)	$x+y$,
(11)	x^5-32y^5.	(*c*)	$x+2y$.

(12) Prove that x^n+1 is divisible by $x+1$ if n be an odd number, but is never divisible by $x-1$, nor by $x+a$, if a be anything different from 1.

(13) Show that y^n-b^n is divisible by $y-b$, but is only divisible by $y+b$ if n be an even number.

(14) If x^m+1 is divisible by $x+1$, prove that $x^{m+2}+1$ is also divisible by $x+1$.

(15) Show that

$$a^n(b-c)+b^n(c-a)+c^n(a-b)$$

has factors $(b-c)(c-a)(a-b)$,

and find the other factors if $n=2$, 3, 4.

(16) Find the remainder when ax^3+bx^2+bx+a is divided by $x-1$; and find the condition that the function be exactly divisible by x^2-1.

(17) If $x^2+2px-3q^2$ be exactly divisible by $x-p$, show that $p=\pm q$.

(18) If $ax^3+3a^2x^2-2ab^2x+a^2b^2$ be divisible by $x-a$, show that $b=\pm 2a$. Is it possible for the expression to be also divisible by $x+a$?

(19) Find the condition that $px^3-(p+q)x^2+qx$ be divisible by x^2-1.

(20) Use the remainder theorem to show that

$$(x-y+z)^2+(y-z+x)^2-(y+z-x)^2+8yz$$

is divisible by $x+y+z$.

Ex. 79. MISCELLANEOUS.

[*Numbers* 1 *to* 10 *are also suitable for oral work.*]

(1) Write down the squares of

(i) $\sqrt{a}+\sqrt{b}$; (ii) $\sqrt{a}-\sqrt{b}$.

(2) Write in full

(i) $\sum_{abc}\frac{a^4}{(a-b)(a-c)}$; (ii) $\sum_{abc}\frac{a^4+b^2c^2}{(a-b)(a-c)}$.

(3) Is $x+y+z$ a factor of $x^4-(y+z)^4$ or of $x^3-(y+z)^3$?

(4) Express with Σ notation (i) the unexpanded and (ii) the expanded form of $(a+b+c+d)^2$.

(5) If $2s = a+b+c$, what are $s-a$, $s-b$, $s-c$?

(6) If $a = x^l$, $b = x^m$, $c = x^n$, what power of x is $a^p b^q \div c^r$?

(7) Find a so that x^2-1 exactly divides $x^6 - 6x^4 + 9x^2 - a$.

(8) Express in the form $\sqrt{a} - \sqrt{b}$:

(i) $\sqrt{10 - 2\sqrt{21}}$; (ii) $\sqrt{11 - 6\sqrt{2}}$.

(9) If $x = 64$ find the values of

$$x^0, \quad x^1, \quad x^{\frac{1}{3}}, \quad (-x)^{\frac{1}{3}}, \quad x^{-\frac{1}{2}}, \quad x^{-\frac{2}{3}}.$$

(10) If $x^2 + 7x + c$ is divisible by $x+4$ what must c be?

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(11) Simplify $96^{\frac{1}{5}} \cdot 243^{\frac{1}{5}} \cdot 75^{\frac{1}{5}} \cdot 3^{\frac{1}{5}\sqrt{5}}$.

(12) Translate into index notation the statements

(i) $\log 19 = 1{\cdot}2788$; (ii) $\text{antilog } 2{\cdot}63 = 426{\cdot}6$.

(13) Prove without the use of tables that

(i) $$\sqrt{\frac{3 \log 1728}{1 + \frac{1}{2}\log(\cdot 36) + \frac{1}{3}\log 8}} = 3,$$

and (ii) $$\frac{\log \sqrt{27} + \log 8 - \frac{1}{2}\log 1000}{\log 1{\cdot}2} = 1\tfrac{1}{2},$$

(14) Divide $1 + x^6$ by $1 - \sqrt{3}x + x^2$.

(15) Simplify $\sqrt{35 - 12\sqrt{6}}$.

(16) Find a, b, c so that for all values of x

$$4x^2 + 17x + 4 = a(x-b)(x-c).$$

(17) Prove that $(b-c)(c-a)(a-b)$

$$= -\Sigma a^2(b-c)$$
$$= \Sigma a(b^2 - c^2)$$
$$= -\Sigma bc(b-c).$$

(18) Simplify

(i) $$\frac{1}{1+\sqrt{1-a}} + \frac{1}{1-\sqrt{1-a}}; \qquad \text{(ii)} \quad \frac{\sqrt{1+a}}{1+\sqrt{1+a}} + \frac{\sqrt{1-a}}{1-\sqrt{1-a}}.$$

(19) Find from the tables

(i) $10^{\cdot 2}$; (ii) $10^{1\cdot 4}$; (iii) $10^{2\cdot 6}$.

(20) Express n^3 as a function of $(n+1)$, *i.e.* in the form $a(n+1)^3+b(n+1)^2+c(n+1)+d$ when a, b, c, d are numbers not depending on n.

---

(21) Bracket the coefficients of the powers of x in the sum of $ax^4+a^2x^3+ax^2+acx$, $2ax^3+2x^2+bx$, bx^3+abx^2+bc, $x^3+2bx^2+b^2x$, cx^2+2cx, and divide the sum by $ax^2+(b+1)x+c$.

(22) Divide $x^4-\frac{1}{x^4}$ by $x+\frac{1}{x}$.

(23) Find the L.C.M. (in factors) of

$$x^2-1,\quad x^2-2x+1,\quad x^3+1,\quad x^2-x+1.$$

(24) Simplify $\dfrac{a^4-x^4}{a+3}\times\dfrac{a^3-x^3}{a^2-2ax+x^2}\times\dfrac{a^2+5a+6}{a^6-x^6}$.

(25) Show that $\sum_{abc}\overline{b+c-a}\,|^2\,a-12abc$ has $a+b+c$ as a factor.

(26) Simplify

$$\frac{a^2-(b-c)^2}{(b+c)^2-a^2}\times\frac{b^2-(c-a)^2}{(c+a)^2-b^2}\times\frac{c^2-(a-b)^2}{(a+b)^2-c^2}.$$

(27) Solve the equations

(i) $12x^2-12x=45$; (ii) $\dfrac{x+2}{x-2}+x=2x-1$.

(28) Find the H.C.F. of

$$x^2+2x-3,\quad x^2-3x+2,\quad x^3+x^2-2x.$$

(29) A and B between them have 444 sheep; A gives B 37 sheep, and then B has three times as many sheep as A. How many had each?

(30) A and B start a business: if A had given £1500 more, what he gave would have been equal to what B gave: if B had given £1500 more, what he gave would have been twice what A gave. Find what each gave.

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(31) Divide out to 5 terms in ascending powers of x

(i) $\dfrac{1+x}{1-x}$; (ii) $\dfrac{1+bx}{1-ax}$.

(32) Multiply $a^{\frac{1}{2}} - b^{\frac{1}{2}}$ by $a^{\frac{3}{2}} + 2a^{\frac{1}{2}}b^{\frac{1}{2}} + 2b^{\frac{3}{2}}$.

(33) Express $\{(x+1)^3 + 3(x+1)^2 - 2(x+1) + 3\}\{4x-1\}$ as a function of $x+1$.

(34) Show that $\left(\frac{3}{\sqrt{x}} - \frac{5}{\sqrt{x}} + 1\right)\left(\frac{3}{\sqrt{x}} - \frac{5}{\sqrt{x}} - 1\right)$ is a rational function of x.

(35) Divide £1000 into two sums, such that, if one be sub-divided into 32 equal parts and the other be sub-divided into 40 equal parts, the difference between the parts in the two cases may be £20.

(36) Find the value of $\sum_{abc}(b-c)(x+b+c)^2$.

(37) Find the remainder when $(x-m)^3 + (x-n)^3 + (m+n)^3$ is divided by $x+m$.

(38) Give the proof, similar to the proof of the usual form of the remainder theorem, that if $f(x)$ be divided by $ax+b$ the remainder is $f\left(-\frac{b}{a}\right)$.

(39) If $x^2 + 6x + b$ and $x^2 + 12x + 3b$ have a common factor, what numerical values can b have, and what is the common factor in each case?

(40) A two-digit number is eight times the sum of its digits. If 9 be added to the number the result is equal to the square of the sum of the digits. Find the number.

Ex. 80. VARIATION.

(1) Translate into symbols the following statements, using a, b for 'constants':

(i) y varies as x; (ii) y varies inversely as x;
(iii) y is proportional to x^2;
(iv) y varies directly as x and inversely as z^2;
(v) z is proportional to x and to y^2;
(vi) y is partly constant and partly proportional to x;
(vii) the square of y varies as the cube of x.

(2) Translate into words the following statements (a, b are constants; x, y, z variables):

(i) $y = ax$; (ii) $y = bx^2$; (iii) $y = a\frac{x^3}{z^2}$;

(iv) $z = a + by$; (v) $z = ax^2y$; (vi) $y^3 = az^2$;

(vii) $y = b\frac{\sqrt{x}}{z}$.

(3) P varies as Q: if $P = 8$, $Q = 10$; find P when $Q = 15$.

(4) If y varies inversely as x and if y is 36 when x is 12, find x when y is 16.

(5) $y = a + bx$ where a, b are constants, y is 12 when x is 2, and 0 when x is 14, find a, b, and the value of y when $x = 20$.

(6) y varies directly as x and $\sqrt{z}$, but inversely as t. If $x = t = 1$ and $z = 49$ then $y = 4$, find y if $x = z = 16$, $t = 2\frac{2}{7}$.

(7) x is proportional to t^2: if x is 1 when t is 2, find x when t is 5.

(8) t is proportional to $\frac{1}{\sqrt{x}}$. When x is 64, t is 7: find x when $t = 16$.

(9) If $t = ax + by$ where a, b are constants, find a, b if $t = 14$ when x, y are each 1, and 10 when $x = 1$, $y = -1$: find also what x is if $y = -4$ and $t = 100$.

(10) Of three artesian wells A, B, C the depths are: A 200 feet, B 365 feet, C 545 feet. The temperatures of the water from these depths are: A 45°, B 51·6°, C 58·8°;

From these observations is it correct to say that (i) the temperature varies as the depth or that (ii) the increase of temperature varies as the increase of depth?

(11) The area of a circle varies as the square of its radius. Given that a circle of radius 5·75 inches has area 103·86 square inches, find the area of a circle whose radius is 6·5 inches.

(12) The diagonal of a cube is proportional to its edge. When the edge is 3 inches the diagonal is 5·2 inches; find the diagonal when the edge is 4 5 inches.

(13) The weight of a coin varies as its thickness and as the square of its radius. One coin has radius half an inch; a second has its thickness double that of the first and its weight less by one-ninth than the weight of the first. Find the radius of the second coin.

(14) A drain-pipe's volume varies as its length and also as the square of its diameter. If a pipe 1 foot in diameter and 100 yards long has the volume 235·7 cubic feet, what is the volume of 80 yards of pipe 10 inches in diameter.

(15) If a weight is hung up by an elastic string, the amount the string stretches varies as the weight and the string's unstretched length, but varies inversely as the square of the string's thickness;

If a 5-lb. weight stretches a string 2 feet long and $\frac{1}{8}$th inch thick to a length 2 feet 6 inches, to what length would a string of the same material but 3 feet long and $\frac{1}{4}$ inch thick be stretched by a weight of 14 lbs.?

(16) The force of the earth's attraction is inversely proportional to the square of the distance from the earth's centre. Taking the earth's radius to be 4000 miles, find how heavy a man of 12 stone weight would become if he could be 200 miles above the earth's surface.

(17) The value of a diamond varies as the square of its weight: a diamond worth £32 is cut into two pieces whose weights are in the ratio 1 : 3. Find the values of the pieces.

(18) The expenses of a workhouse are partly constant and partly proportional to the number of inmates. With 150 inmates the expenses are £900, with 120 inmates £750. Find the expenses for 200 inmates.

(19) If income tax were proportional to the square of the income and if an income of £100 paid £1, show that a man whose (nominal) income was £10,000 would be a pauper.

Which would really be the richer, a man whose (nominal) income was £800 or one whose (nominal) income was £9000, and by how much?

Ex. 81. EQUAL RATIOS (OR FRACTIONS).

(1) If $\frac{a}{b}=\frac{c}{d}$, prove that

(i) $\frac{a+b}{b}=\frac{c+d}{d}$; (ii) $\frac{a-b}{b}=\frac{c-d}{d}$; (iii) $\frac{a+c}{c}=\frac{b+d}{d}$;

(iv) $a^2d^2=b^2c^2$; (v) $\frac{a}{b}=\frac{4a+5c}{4b+5d}$.

(2) If $a:b=c:d$, prove that

(i) $la^2+mc^2 : lb^2+md^2=a^2:b^2$;

(ii) $\sqrt{a+2c}:\sqrt{b+2d}=\sqrt{2a+c}:\sqrt{2b+d}$.

(3) If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, prove that

(i) $\frac{x+y+z}{a+b+c}=\frac{px+qy+rz}{pa+qb+rc}=\frac{ax+by+cz}{a^2+b^2+c^2}=\sqrt{\frac{x^2+y^2+z^2}{a^2+b^2+c^2}}=\frac{x}{a}$;

(ii) $\frac{4ax^3+y^3+3cz^3}{4a^4+b^3+3c^4}=\sqrt{\frac{x^6+y^6-z^6}{a^6+b^6-c^6}}$.

(4) If $\frac{x}{p-q}=\frac{y}{q-r}=\frac{z}{r-p}$, prove that $x+y+z=0$.

(5) If $\frac{x}{2a-b-c}=\frac{y}{2b-c-a}=\frac{z}{2c-a-b}$,

prove that $x+y+z=0$.

(6) If $\frac{p}{2a+b}=\frac{q}{a+b+c}=\frac{r}{b+2c}$, prove that $p+r=2q$.

(7) If $\frac{x^2}{a^2-b^2}=\frac{y^2}{b^2-c^2}=\frac{z^2}{c^2-a^2}$,

then $c^2x^2+a^2y^2+b^2z^2=0$ and $x^2+y^2+z^2=0$.

(8) If $\frac{x}{b-2c}=\frac{y}{c-2a}=\frac{z}{a-2b}=\frac{w}{a+b+c}$, then $x+y+z+w=0$.

(9) If $\frac{x}{z}=\frac{x-y}{y-z}$, prove that $\frac{2}{y}=\frac{1}{x}+\frac{1}{z}$.

(10) If $a:b=b:c$, prove $\frac{a-b}{b-c}=\frac{b}{c}$, $\frac{a}{c}=\frac{a^2}{b^2}=\frac{a^2+b^2}{b^2+c^2}$.

Ex. 82. **H. C. F.**

Find the highest common factor of

(1) $3x^2+16x-35$, $5x^2+33x-14$.

(2) $3x^2-4x-15$, $x^3-2x^2-15x+36$.

(3) x^3+6x^2-8x-7, $x^3+8x^2+10x+21$.

(4) $4y^4-5y^2+1$, $4y^4+4y^3+y^2-1$.

(5) a^3+5a^2+8a+4, $a^3+6a^2+11a+6$.

(6) $6x^3+7x^2y-22xy^2-5y^3$, $15x^3-14x^2y-13xy^2-2y^3$.

Find the lowest common multiple of

(7) $9x^4-4x^2+4x-1$, $9x^4-12x^3+4x^2-1$.

(8) x^2-4x+3, $x^4-10x^3+37x^2-50x+22$.

(9) $2x^3-x^2+4x+7$, $6x^4-11x^3+28x^2-13x+14$.

(10) x^3-8x+3, x^6+3x^5+x+3.

(11) a^3-a^2+a+14, $a^3-5a^2+13a-14$.

Find the highest common factor of

(12) x^3-5x^2+9x-9, $3x^3-5x^2-7x-15$.

(13) $27x^5-13x^2-14$, $3x^5-x^3-2x$.

(14) $3x^3+2x^2-18x+8$, $6x^3-11x^2+13x-12$.

Reduce to their lowest terms

(15) $\dfrac{x^4-x^3-7x^2+13x-6}{x^4+x^3-4x^2+5x-3}$.

(16) $\dfrac{6x^3-23x^2+26x-8}{2x^3-13x^2+27x-18}$.

(17) $\dfrac{x^4-x^3+3x^2-2x+20}{2x^4+x^3+6x^2-4x+16}$.

(18) $\dfrac{15y^3-26y^2+29y-28}{21y^3-22y^2-23y+20}$.

Find the highest common factor of

(19) $2x^5-4x^4+8x^3-12x^2+6x$, $3x^5-3x^4-6x^3+9x^2-3x$.

(20) $x^5-209x+56$, $56x^5-209x^4+1$.

(21) $6x^4-13x^3+19x^2-7x-5$,
$10x^5-19x^4+41x^3-33x^2+36x-35$.

Ex. 83. **FRACTIONS.**

Simplify:

(1) $\frac{1}{x+5}+\frac{1}{x+3}$.

(2) $\frac{1}{x+7}-\frac{1}{x+9}$.

(3) $\frac{2}{3x+1}-\frac{1}{x+3}$.

(4) $\frac{3x}{4x+2}+\frac{x+1}{2x+6}$.

(5) $\frac{a}{a+b}-\frac{b}{a-b}$.

(6) $\frac{2x}{(x-2)^2}+\frac{1}{(x-2)}$.

(7) $\frac{2a-b}{a^2-b^2}-\frac{3}{2a+2b}$.

(8) $\frac{1}{x^2-x-2}+\frac{1}{x^2-4}$.

(9) $\frac{2}{x^2-7x+12}-\frac{1}{x^2-3x-4}$.

(10) $\frac{2x+1}{(x-1)^2}-\frac{x}{x^2-1}$.

(11) $\frac{1}{x-y}+\frac{2}{x+y}-\frac{x}{2(x^2-y^2)}$.

(12) $\frac{3a}{a+b}-\frac{2b}{3(a-b)}+\frac{4(a^2+b^2)}{3a^2-3b^2}$.

(13) $\frac{1}{x+y}-\frac{2x}{x^2-2xy-3y^2}+\frac{x+3y}{x^2-9y^2}$.

(14) $\frac{1}{x+1}-\frac{2}{x+2}+\frac{1}{x+3}$.

(15) $\frac{3}{5x-1}+\frac{1}{4x-3}-\frac{2}{3x-2}$.

(16) $\frac{4a}{2x+1}-\frac{2a}{x+1}+\frac{a}{x-4}$.

(17) $\frac{1}{2-a}-\frac{2}{3-a}+\frac{1}{4-a}$.

(18) $\frac{3a+2x}{3a-2x}-\frac{3a-2x}{3a+2x}+\frac{16x^2}{4x^2-9a^2}$.

(19) $\frac{a-b}{b}+\frac{2a}{a-b}-\frac{a^3+a^2b}{a^2b-b^3}$.

(20) $\frac{x}{x+3a}-\frac{x}{x-3a}+\frac{4ax}{x^2-9a^2}-\frac{4ax}{x^2+9a^2}$.

(21) $\frac{2x}{3(x^2-y^2)}+\frac{2(x+y)}{x^2-xy}-\frac{1}{3(x+y)}$.

(22) $\frac{a-b}{a-2b}-\frac{2(a+b)}{a+3b}+\frac{a-b}{a+4b}$.

(23) $\left\{\frac{a}{a-b}+\frac{b}{a+b}\right\}+\left\{\frac{a}{a+b}-\frac{b}{a-b}\right\}$.

(24) $\frac{1}{x-1}-\frac{4}{1-x}-\frac{8}{1+x}+\frac{3x+7}{x^2-1}$.

(25) $\frac{1}{b+3a}-\frac{1}{3a-b}+\frac{6a}{b^2-9a^2}$.

(26) $\dfrac{a^4+a^2b^2+b^4}{a^6-b^6}+\dfrac{a+b}{a^3-b^3}-\dfrac{a-b}{a^3+b^3}-\dfrac{1}{a^2-b^2}$.

(27) $\dfrac{a+b}{ax+by}+\dfrac{a-b}{ax-by}+\dfrac{2(a^2x+b^2y)}{a^2x^2+b^2y^2}$.

(28) $\dfrac{1}{x^2-3x+2}-\dfrac{2x}{x^2-4x+3}+\dfrac{1}{x^2-5x+6}$.

(29) $\dfrac{5x+4}{x-2}-\dfrac{3x-2}{x-3}-\dfrac{x^2-2x-17}{x^2-5x+6}$.

(30) $\dfrac{2}{x^2+x-2}+\dfrac{3}{4-x^2}+\dfrac{5}{x^2-x-2}$.

(31) $\dfrac{1}{x^2-3x+2}+\dfrac{2x}{x^2-4x+3}+\dfrac{1}{x^2-5x+6}$.

(32) $\left\{\dfrac{a+bx}{a-bx}+\dfrac{b+ax}{b-ax}\right\}\div\left\{\dfrac{a+bx}{a-bx}-\dfrac{b+ax}{b-ax}\right\}$.

(33) $\left\{\dfrac{x+2y}{x+y}+\dfrac{x}{y}\right\}\div\left\{\dfrac{x+2y}{y}-\dfrac{x}{x+y}\right\}$.

(34) $\left\{1-\dfrac{a^2+b^2-c^2-d^2}{2(ab+cd)}\right\}\left\{\dfrac{(a+c)(b+d)-bc-ad}{a+c+d-b}\right\}$.

(35) $\left\{\dfrac{x}{x-y}-\dfrac{y}{x}\right\}\div\left\{\dfrac{x}{y}+\dfrac{y}{x+y}\right\}$.

(36) $\dfrac{\dfrac{1}{1+x}+\dfrac{x}{1-x}}{\dfrac{1}{1-x}-\dfrac{x}{1+x}}$.

(37) $\dfrac{ab\left(\dfrac{1}{a}-\dfrac{1}{b}\right)}{\dfrac{a}{b}+\dfrac{b}{a}+2}\div\dfrac{\dfrac{a}{b}-\dfrac{b}{a}}{\left(\dfrac{1}{a}+\dfrac{1}{b}\right)^2}$.

(38) $\dfrac{\dfrac{a}{a-b}+\dfrac{b}{a+b}}{\dfrac{a}{a+b}-\dfrac{b}{a-b}}$.

(39) $\dfrac{1-\dfrac{2xy}{x^2+y^2}}{\dfrac{x^3-y^3}{x-y}-3xy}$.

(40) $\left(y+\dfrac{m-xy}{x-y}\right)\left(x-\dfrac{m-xy}{x-y}\right)+\left(\dfrac{m-xy}{x-y}\right)^2$.

(41) $\dfrac{a^2(b-c)}{(a+b)(a+c)}+\dfrac{b^2(c-a)}{(b+c)(b+a)}+\dfrac{c^2(a-b)}{(c+a)(c+b)}$.

(42) $\dfrac{(a+d)bc}{(a-b)(a-c)}+\dfrac{(b+d)ca}{(b-c)(b-a)}+\dfrac{(c+d)ab}{(c-a)(c-b)}$.

Simplify :

(43) $\dfrac{2a^2+bc}{(c-a)(a-b)}+\dfrac{2b^2+ca}{(a-b)(b-c)}+\dfrac{2c^2+ab}{(b-c)(c-a)}$.

(44) $\dfrac{(a-b)^2-(b-c)^2}{a^2+ab-bc-c^2}+\dfrac{(b-c)^2-(c-a)^2}{b^2+bc-ca-a^2}+\dfrac{(c-a)^2-(a-b)^2}{c^2+ca-ab-b^2}$.

(45) $\left\{\dfrac{b+c-2a}{b-c}+\dfrac{c+a-2b}{c-a}+\dfrac{a+b-2c}{a-b}\right\}$
$\times\left\{\dfrac{(b-c)bc+(c-a)ca+(a-b)ab}{(b+c-2a)(c+a-2b)(a+b-2c)}\right\}$.

(46) $\sum_{abc}\dfrac{1}{(a-b)(a-c)}$.

(47) $\sum_{xyz}\dfrac{1}{y-z}$.

(48) $\sum_{xyz}\dfrac{x}{y-z}$.

(49) $\sum_{abc}\dfrac{a^2}{(a-b)(a-c)}$.

(50) $\sum_{abc}\dfrac{a+x}{(a+b)(a+c)}$.

(51) $\sum_{xyz}\dfrac{x^2+ax+a^2}{(x-y)(x-z)}$.

(52) $\dfrac{\sum_{abc}a^2\left(\frac{1}{b}-\frac{1}{c}\right)}{\sum a\left(\frac{1}{b}-\frac{1}{c}\right)}$.

(53) $\sum_{abc}\dfrac{2a-b-c}{(x-a)(a-b)(a-c)}$.

(54) $\sum_{xyz}\dfrac{x(s^2-yz)}{(s-y)(s-z)}$, where $s=\Sigma x$.

Ex. 84. LITERAL EQUATIONS.

(See also the next Exercise.)

Solve the equations for x, or x and y :

(1) $px+q=rx+t$.

(2) $ax+b^2=a^2-bx$.

(3) $\dfrac{x}{a}+\dfrac{x}{b}=1$.

(4) $(x+a)(x+b)-x^2=3ab$.

(5) $3x^2-a^2=3a^2-x^2$.

(6) $x^2+ab=x(a+b)$.

(7) $\left.\begin{aligned}ax+by&=a^2+b^2,\\x+y&=a+b.\end{aligned}\right\}$

(8) $\left.\begin{aligned}px+qy&=r,\\x+y&=0.\end{aligned}\right\}$

(9) $\left.\begin{aligned}lx+my&=m,\\mx+ly&=l.\end{aligned}\right\}$

(10) $\left.\begin{aligned}x-y&=a,\\x^2+2b^2&=3by+3ab.\end{aligned}\right\}$

Ex. 85. EQUATIONS INVOLVING FRACTIONS.

Solve the equations:

(1) $\dfrac{2}{1-5x}=\dfrac{5}{1-2x}$.

(2) $\dfrac{3x}{x+6}-\dfrac{x-1}{2x+1}=1$.

(3) $\dfrac{3x-4}{2x-3}-\dfrac{2x+3}{3x-7}=-\dfrac{3}{5}$.

(4) $\dfrac{4x^3+4x^2+8x+1}{2x^2+2x+3}=\dfrac{2x^2+2x+1}{x+1}$.

(5) $\dfrac{x+1}{x^2+x+1}+\dfrac{x^2-x+1}{x-1}=\dfrac{x^2}{x+1}$.

(6) $\dfrac{2x+9}{2x-9}-\dfrac{3x+13}{3x-13}=\left\{\dfrac{x+4}{x-4}-\dfrac{4x+15}{4x-15}\right\}$.

(7) $\dfrac{2(x+4)}{x-5}-\dfrac{14-x}{x-2}=4\frac{2}{3}$.

(8) $\dfrac{3(2x+1)}{x-2}-\dfrac{13-3x}{x-1}=19$.

(9) $\left.\begin{aligned}&\dfrac{x}{x+y}+\dfrac{y}{x-y}=\dfrac{5}{4},\\&\dfrac{x}{x-y}+\dfrac{y}{x+y}=\dfrac{14}{x^2-y^2}.\end{aligned}\right\}$

(10) $\dfrac{x-a}{x+a}+\dfrac{x^2+a^2}{x^2-a^2}=\dfrac{x+a}{x-a}+\dfrac{x^2-a^2}{x^2+a^2}$.

(11) $\dfrac{x-a^2}{b}-\dfrac{x-b^2}{a+b}+\dfrac{x-3ab}{a-b}=a$.

(12) $\dfrac{(x+a)(x+b)}{x+a+b}=\dfrac{(x+c)(x+d)}{x+c+d}$.

(13) $\left.\begin{aligned}&(a+b)x-(a-b)y=4ab,\\&(a+b)x+(a-b)y=2(a^2+b^2).\end{aligned}\right\}$

(14) $\left\{\dfrac{x+2a}{x+2b}\right\}^{\frac{1}{3}}=\dfrac{x+a}{x+b}$.

(15) $\dfrac{a-bx}{a+bx}+\dfrac{ax-b}{ax+b}=\dfrac{2(a-b)}{a+b}$.

(16) $\dfrac{x-a-b}{x-c-d}=\dfrac{a+b}{c+d}$.

(17) $\dfrac{1}{x+a}+\dfrac{1}{x-b}=\dfrac{1}{x-a}+\dfrac{1}{x+b}$.

(18) $x^2+\left(\dfrac{a^2}{b}+\dfrac{b^2}{a}\right)x+ab=0$.

(19) $\left.\begin{aligned}&cy+bz=b+c-a,\\&az+cx=c+a-b,\\&bx+ay=a+b-c.\end{aligned}\right\}$

(20) $\left.\begin{aligned}&ax+by=ac,\\&cy+dz=ad,\\&bx+ay+cz=c(a+b).\end{aligned}\right\}$

Ex. 86. **SQUARE ROOT.**

Find the square root of

(1) x^2+6x+9. (2) $x^2-4xy+4y^2$.

(3) $16a^2+24ab+9b^2$. (4) $25x^2-10xy+y^2$.

(5) $1-14ab+49a^2b^2$. (6) $x^4+16x^2y^2+64y^4$.

(7) $x^4-6x^3+17x^2-24x+16$.

(8) $9a^4-12a^3b+34a^2b^2-20ab^3+25b^4$.

(9) $4x^6-12x^5-7x^4+44x^3-14x^2-40x+25$.

(10) $x^6-4x^5+6x^3+8x^2+4x+1$.

(11) $49y^6-112y^5+92y^4-46y^3+20y^2-4y+1$.

(12) $25x^6-30x^5y+9x^4y^2+10x^3y^2-6x^2y^3+y^4$.

(13) $a^6-4a^5b+6a^2b^2-10a^3b^3+13a^2b^4-6ab^5+9b^6$.

(14) $9x^6-6x^4+24x^3+x^2-8x+16$.

(15) $29x^4y^2+16y^6-12x^5y-46x^3y^3+4x^6-40xy^5+49x^2y^4$.

(16) $4a^4-12a+4+9a^{-2}-6a^{-3}+a^{-4}$.

(17) $\frac{1}{4}-2x+7x^2-\frac{37}{3}x^3+\frac{31}{3}x^4-2x^5+\frac{1}{9}x^6$.

(18) $a^{-\frac{1}{3}}+9a^{\frac{2}{3}}c^{-2}+4b^{-\frac{2}{3}}c^{\frac{4}{3}}+12a^{\frac{1}{3}}b^{-\frac{1}{3}}c^{-\frac{1}{3}}+4a^{-\frac{2}{3}}b^{-\frac{1}{3}}c^{\frac{2}{3}}+6a^{-\frac{1}{3}}c^{-1}$.

(19) By extracting the square root of

$$x^6+6x^5+5x^4-10x^3+11x^2-7x+3,$$

find two values of x which will make this quantity a perfect square.

[Ex. 1 to 6 above would naturally be solved by inspection. Most of the others can be readily solved by the method of 'indeterminate coefficients,' as suggested in the following examples, in which, however, the rule may be used if preferred.]

(20) If the square root of $x^4-8x^3+30x^2-56x+49$ be x^2+ax+7, show that $a=-4$.

(21) If the square root of $x^4-14x^3+ax^2+bx+4$ be x^2-cx+2, find a, b, c.

(22) If the square root of $x^6+4x^5+2x^4-2x^3+5x^2-2x+1$ be x^3+ax^2+bx+1, find a and b.

If it is known that the expression is a perfect square, how many of its coefficients are necessary in order that its square root may be found?

How many conditions must an integral function of x of the sixth degree satisfy in order to be a perfect square?

(23) Find the square root of $4x^4+20x^2-3-\frac{70}{x^2}+\frac{49}{x^4}$.

(24) Of what form will the square root of

$$x^4+6x^3+9x^2-2x^2y^2-axy^2+y^4$$

be? Find the square root and the value of a that the expression may be a perfect square.

Ex. 87. ARITHMETIC PROGRESSION.

Oral.

For each of the following series state

(i) the common difference; (ii) the last term;
(iii) the average of the first and last terms; (iv) the sum:

(1) 12, 14, 16,... 15 terms. (2) 14, 17, 20,... 21 terms.
(3) 1, 5, 9,... 26 terms. (4) 100, 99, 98,... 20 terms.
(5) $-6, -7, -8,...$ 10 terms. (6) $-7, -3, 1,...$ 16 terms.
(7) $10, 10\frac{1}{2}, 11,...$ 18 terms. (8) $20, 19\frac{1}{3}, 18\frac{2}{3},...$ 31 terms.
(9) $-1, -\frac{2}{3}, -\frac{1}{3},...$ 43 terms. (10) $4, -1, -6,...$ 51 terms.

For each of the following series state

(i) the 2nd term; (ii) the 3rd term;
(iii) the number of terms; (iv) the sum:

(11) First term, 100; common difference, 3; last term, 310.
(12) „ 40; „ -4; „ 4.
(13) „ 20; „ $\frac{1}{2}$; „ 30.
(14) The even numbers from 100 to 200.
(15) The odd numbers between 300 and 500.

For each of the following series state the n^{th} term:

(16) 1, 2, 3,.... (17) 1, 3, 5,.... (18) 2, 4, 6,....

(19) 3, 6, 9,.... (20) 1, 4, 7,.... (21) 10, 15, 20,....

(22) 1, 0, -1,.... (23) x, $x+1$, $x+2$,...

(24) a, $a+d$, $a+2d$,....

What is the arithmetic mean between

(25) 7, 9. (26) 14, 26. (27) 20, 21. (28) 31, 42.

(29) 15, 27. (30) 20, x. (31) $20-x$, $20+x$. (33) a, b.

Between each of the following pairs of numbers two arithmetic means are inserted. Find them:

(34) 10, 13. (35) 40, 46. (36) 54, 66.

(37) 30, 31. (38) 10, $10+3x$. (39) x, $x+3y$.

(40) a, b. (41) $a-3d$, $a+3d$.

Ex. 88.

Find the last term and the sum of the following series:

(1) 20, 31, 42, ... to 16 terms. (2) -3, -1, 1, ... to 40 terms.

(3) 30, 22, 14, ... to 9 terms. (4) 1, 2, 3, ... to n terms.

(5) 1, 3, 5, ... to n terms. (6) x, $4x$, $7x$, ... to n terms.

(7) $2\frac{1}{4}$, $5\frac{1}{2}$, $8\frac{3}{4}$, ... to n terms. (8) $\frac{3a}{7}$, $\frac{9a}{14}$, $\frac{6a}{7}$, ... to n terms.

(9) If a, b are the first two terms of an A.P., what is the n^{th} term?

(10) By going through the general process of adding the series itself to the series reversed, sum the series a, $a+x$, ... $a+\overline{n-1}.x$, $a+nx$. How many terms are there in the series?

(11) By going through the general process of adding the series itself to the series reversed, find the sum of $2n+1$ terms in A.P., of which the three middle ones are $a-d$, a, $a+d$; and show that this sum does not depend on d.

(12) If an A.P. has an even number of terms of which $a-d$, $a+d$ are the two middle ones, show that the sum does not depend on d, and find the sum to n terms.

(13) The series 58, 61, 63, 66, ... 93, 96, 98, 101 has the differences between its terms alternately 3 and 2. By adding the series itself to the series reversed, find its sum. In what other way could the sum be found without writing down all the terms of the series?

(14) The series 70, 72, 73, 75, ... 94, 96, 97 has the differences between its terms alternately 2 and 1. By adding the series itself to the series reversed, find its sum.

(15) Find the arithmetic mean between

(i) $a+b$, $a-b$; (ii) $\frac{1}{a}$, $\frac{1}{b}$; (iii) $\frac{x^2+y^2}{x+y}$, $\frac{2xy}{x+y}$.

(16) Insert 5 arithmetic means between $\frac{1}{2}$ and $-\frac{1}{2}$.

(17) Insert 16 arithmetic means between 19·2 and 5·6.

(18) Insert n arithmetic means between a and b.

Find the series in which

(19) The 1st term is 18, and the 30th term -40.

(20) The 26th term is 100, and the 50th term 196.

(21) The 15th term is -50, and the 25th term 0.

(22) The 32nd term is 28, and the 50th term 42.

How many terms must be taken

(23) Of the series 20, 23, 26, ... to make 385.

(24) Of the series 80, 68, 56, ... to make 28.

(25) Of the series -5, -1, 3, ... to make 897.

(26) Of the series 10, $9\frac{1}{2}$, 9, ... to make 105.

(27) Of the series 28, 26, 24, ... to make 180?

Explain the double answers in questions 26, 27.

(28) Construct an arithmetic progression of which the sum to two terms is the same as the sum to 7 terms, and whose common difference is 3.

Ex. 89. GEOMETRIC PROGRESSION.

Oral.

For each of the following series state

(i) the common ratio; (ii) the fourth term; (iii) the n^{th} term:

(1) 1, 2, 4, ... (2) 4, 8, 16, ... (3) 1, 3, 9, ...

(4) 2, 6, 18, ... (5) $1, x, x^2, \ldots$. (6) $a, ar, ar^2, \ldots$.

(7) $x^2, x^4, x^6, \ldots$. (8) $1, \frac{1}{x}, \frac{1}{x^2}, \ldots$. (9) $ab^2, a^2b^3, a^3b^4, \ldots$

For each of the following series state

(i) the second term; (ii) the n^{th} term:

(10) First term 7, common ratio 2.

(11) First term 16, common ratio $\frac{1}{2}$.

(12) First term 1, common ratio x.

(13) First term a, common ratio x^3.

What is the geometric mean between

(14) 1 and 81. (15) 5 and 20. (16) a and ar^2.

(17) $\frac{1}{x}$ and x. (18) ax and ax^5. (20) $18a$ and $\frac{a}{2}$.

For the following series state

(i) the sum to n terms; (ii) the sum to infinity if possible:

(21) 4, 8, 16, ... (22) 4, 2, 1, ... (23) $1, \frac{1}{3}, \frac{1}{9}, \ldots$.

(24) 12, 18, 27, ... (25) 27, 18, 12, ... (26) $1, \frac{1}{2}, \frac{1}{4}, \ldots$.

(27) $27, -18, 12, \ldots$. (28) $-1, \frac{1}{4}, -\frac{1}{16}, \ldots$. (29) $1, x, x^2, \ldots$.

(30) $a, -ab, ab^2, \ldots$. (31) $x, -\frac{x^2}{2}, \frac{x^2}{4}, \ldots$ (32) $x^4, x^3, x^2, \ldots$.

State the common ratio in each of the following G.P.'s:

(33) $\cdot\dot{4}$. (34) $\cdot\dot{6}\dot{3}$. (35) $\cdot02\dot{1}\dot{8}$.

Ex. 90.

Find the n^{th} term of each of the following series:

(1) $\frac{8}{9}, \frac{4}{3}, 2, \ldots.$ (2) $2, -\frac{2}{3}, \frac{2}{9}, \ldots.$

(3) $\frac{a^5}{x^5}, \frac{a^4}{x^4}, \frac{a^3}{x^3}, \ldots.$ (4) $a, \sqrt{x}, \frac{x}{a}, \ldots.$

Find the sum to 10 terms of each of the following series:

(5) $5, \frac{5}{2}, \frac{5}{4}, \ldots.$ (6) $28, -14, 7, \ldots.$

(7) $243, 81, 27, \ldots.$ (8) $243, -81, 27, \ldots.$

By going through the general process of subtracting the series × common ratio from the series itself, find the sum of

(9) $1 + x + x^2 + \ldots + x^n$.

(10) $1 - 2x + 4x^2 - \ldots$ to 20 terms.

Apply the same method to sum the following series:

(11) $1 + 2x + 3x^2 + \ldots$ to 20 terms.

(12) $2 + 7x + 12x^2 + \ldots$ to 30 terms.

(13) $a + 2ax + 3ax^2 + \ldots$ to n terms.

(14) $a - 4ax + 7ax^2 - \ldots$ to 25 terms.

Find by summing the series to infinity the values of

(15) $\cdot\dot{2}$. (16) $\cdot\dot{7}\dot{1}$. (17) $\cdot 61\dot{6}$.

(18) $2\cdot\dot{0}\dot{5}$. (19) $1\cdot 24\dot{0}1\dot{6}$.

Using logarithms, sum to 100 terms each of the following series:

(20) $1 + \frac{3}{2} + \frac{9}{4} + \ldots.$ (21) $1 + \frac{1}{2} + \frac{1}{4} + \ldots.$

(22) $1 + 1\cdot 2 + 1\cdot 44 + \ldots.$ (23) $1 - \cdot 7 + \cdot 49 - \ldots.$

(24) A horse dealer offered to sell a horse for the price of the nails in his shoes, to be paid thus: ¼d. for the 1st nail, ½d. for the 2nd nail, 1d. for the 3rd, and so on—twice as much for any nail as for the previous one. The would-be purchaser accepted. If there were 24 nails, how much had he to pay?

Ex. 91. HARMONIC PROGRESSION

(and miscellaneous questions on the progressions).

Oral.

For each of the following series state (i) the corresponding A.P.; (ii) the n^{th} term of that A.P.; (iii) the n^{th} term of the series itself:

(1) $1, \frac{1}{2}, \frac{1}{3}, \ldots$ (2) $1, \frac{1}{3}, \frac{1}{5}, \ldots$ (3) $\frac{1}{7}, \frac{1}{11}, \frac{1}{15}, \ldots$

(4) $\frac{2}{5}, \frac{1}{2}, \frac{2}{3}, \ldots$ (5) $\frac{4}{5}, \frac{2}{3}, \frac{4}{7}, \ldots$ (6) $\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \ldots$

What is the harmonic mean between

(7) $\frac{1}{9}$ and $\frac{1}{11}$. (8) $\frac{1}{16}$ and $\frac{1}{26}$. (9) $\frac{3}{7}$ and $\frac{1}{3}$.

(10) $\frac{1}{a-d}$ and $\frac{1}{a+d}$. (11) $\frac{1}{x}$ and $\frac{1}{y}$. (12) a and b.

To which, if any, of the progressions do the following series belong:

(13) $2, 6, 18, \ldots$ (14) $\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$ (15) $\frac{1}{4}, \frac{1}{2}, 1, \ldots$

(16) $\frac{1}{4}, \frac{1}{2}, 2, \ldots$ (17) $\frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \ldots$ (18) $2, 1\frac{1}{2}, 1\frac{1}{5}, \ldots$

(19) $2, 1\frac{1}{2}, 1\frac{1}{8}, \ldots$ (20) $3, 1, -1, \ldots$

(21) Give the usual methods of representing three terms (i) in A.P., (ii) in G.P., (iii) in H.P.

(22) If it is desired to make their sum as simple as possible, how are (i) 3 terms, (ii) 4 terms in A.P. represented?

(23) If it is desired to make their product as simple as possible, how are (i) 3 terms, (ii) 4 terms in G.P. represented?

(24) What is the condition that a, b, c be in A.P.?

(25) What is the condition that a, b, c be in G.P.?

(26) What is the condition that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ be in A.P.?

Ex. 92.

(1) Given that $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P.; prove that $\frac{a}{c}=\frac{a-b}{b-c}$.

(2) Given $\frac{a}{c}=\frac{a-b}{b-c}$; prove $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ in A.P.

(3) If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in A.P., prove $b=\frac{2ac}{a+c}$, and conversely.

Find the n^{th} term of each of the series:

(4) $\frac{1}{23}, \frac{1}{27}, \frac{1}{31}, \ldots$ (5) $\frac{1}{x}, \frac{1}{2x+1}, \frac{1}{3x+2}, \ldots$

Insert three harmonic means between

(6) $\frac{1}{2}$ and $\frac{1}{3}$. (7) 2 and 3.

(8) Find the third term of (i) the A.P., (ii) the G.P., (iii) the H.P. of which x, y are the first two terms.

(9) If A, G, H are the arithmetic, geometric, and harmonic means between a and b, express them in terms of a and b, and prove that $G^2=A.H$; also that $A>G>H$.

(10) If a, b, c are in A.P.; a, b, d in G.P.; find $d:c$ in terms of a and b.

(11) The sum of three numbers in A.P. is 36, and the sum of their squares is 440; find the numbers.

(12) The sum of four numbers in A.P. is 88, and the product of the means exceeds the product of the extremes by 32; find the numbers.

(13) The product of three numbers in G.P. is 27, and the sum of the first and last is $12\frac{1}{4}$; find the numbers.

(14) 20 stones are placed on the ground at intervals of 6 yards apart. A runner has to start from a basket 10 yards from the first one, pick up the stones, and bring them back to the basket one by one. How far has he to run altogether?

(15) A and B start together to bicycle along a certain road. A goes at a uniform pace of 10 miles an hour. B goes at 16 mi./hr. for the first hour, 13 mi./hr. for the second hour, and so on, 3 mi./hr. less each hour. After how many hours will A have overtaken B?

(16) In an increasing A.P. of positive terms show that the ratio of each term to the preceding term continually diminishes.

(17) If $a, b, c, d \ldots$ are in G.P., show that a^2-b^2, b^2-c^2, $c^2-d^2, \ldots$ are also in G.P.; and that if summation to infinity be possible the sum of the latter series is a^2.

(18) If a, b, c are in H.P., prove that $\frac{b+c}{a}, \frac{c+a}{b}, \frac{a+b}{c}$ are in A.P.

(19) If $a, b, c, d, \ldots$ are in G.P., so are $a+b, b+c, c+d, \ldots$ and $ab, bc, cd, \ldots$.

Prove these statements and generalize them.

(20) If $b+c, c+a, a+b$ are in H.P., then a^2, b^2, c^2 are in A.P. but if $b+c, c+a, a+b$ are in A.P., then a, b, c are in A.P.

(21) If a, b, c are in G.P. show that the ratio of the harmonic mean of b and c to the harmonic mean of a and b is $b : a$.

Ex. 93. PERMUTATIONS AND COMBINATIONS.

(1) The letters a, b, c, d are to be arranged in four places; write down

(i) the different ways of filling the first place.

(ii) the different ways of filling the first two places.

(iii) the number of different ways in which the first three places can be filled.

(2) If there are 5 letters a, b, c, d, e, write down the different ways of filling two places.

(3) There are 8 steamers running from Dublin to Holyhead and back. In how many ways is it possible to cross and return by a different steamer?

(4) Write out the permutations of the letters of the word *fate* taken all together.

How will the number be reduced if the first letter must be

(i) a consonant; (ii) the consonant f?

(5) Find the values of

(i) ${}^{100}P_2$; (ii) ${}^{7}P_6$; (iii) ${}^{20}P_3$.

(6) Find the number of permutations all together of the letters of the words

(i) *crown*; (ii) *cloister*.

(7) Prove that

(i) ${}^{n}P_n = {}^{n}P_r \,.\, \overline{{}^{n-r}P_{n-r}}$; (ii) ${}^{n}P_n = n \,.\, \overline{{}^{n-1}P_{n-1}}$.

(8) Show that ${}^{n}P_r$ increases as r increases up to $n-1$. If ${}^{n}P_r = {}^{n}P_{r-1}$ what must r be?

(9) Evaluate $\frac{12!}{9!} \cdot \frac{8!}{7!} \cdot \frac{2!}{5!}$.

(10) Find the value of

(i) $\frac{n!}{\overline{n-3}!}$. (ii) $\frac{n!}{n(n-1)(n-2)(n-3)}$.

(11) Prove that $\frac{2n!}{1 \,.\, 3 \,.\, 5 \,.\, \ldots\, \overline{2n-1}} = 2^n \,.\, n!$.

Ex. 94.

(1) Write down (i) the combinations, (ii) the permutations of the letters a, b, c, d taken 3 at a time, arranging them so as to show that 6 permutations correspond to each combination. Of what general result is this a particular case?

(2) Do the same as in question (1) for the five letters a, b, c, d, e taken two at a time. Also write down the combinations of these five letters three at a time, and arrange so as to show why ${}^{5}C_2 = {}^{5}C_3$.

(3) Calculate ${}^{20}C_2$, ${}^{10}C_3$, ${}^{11}C_9$, ${}^{14}C_{11}$.

(4) Out of 14 players, in how many ways can a cricket eleven be chosen

(i) if no places in the team have been filled;

(ii) if 8 places have been already filled?

(5) If ${}^nC_3 = 6 \cdot \overline{{}^{n-1}C_2}$ find n.

(6) If ${}^nC_4 = 5 \cdot \overline{{}^{n-2}C_3}$ find n.

(7) In how many ways can a committee of 2 masters and 3 boys be chosen from 7 masters and 12 boys?

(8) How many words each of 4 consonants and 3 vowels, with the vowels in the even places, can be made from the letters of the words

(i) *equivocal*; (ii) *facetiously*?

(9) Write out the values of

(i) ${}^7C_r$ for $r = 1, 2, 3, \ldots 6$,

(ii) ${}^8C_r$ for $r = 1, 2, 3, \ldots 7$,

and hence explain in general terms how ${}^nC_r$ changes for values $r = 1, 2, \ldots n-1$, pointing out when there is one value of r for which ${}^nC_r$ is greatest, and when there are two such values.

Ex. 95.

(1) Write down the permutations of a, b, c, d taken all together, and arrange so as to show the change in their number if a, b, c, d are replaced by a, b, b, b.

(2) Find the number of permutations of the letters of the following words taken all together:

(i) *Laocoon*, (ii) *parallelogram*, (iii) *Mississippi*.

(3) In how many ways can 3 numbers be selected from 50 on a combination lock, repetition being possible?

Find the chance of guessing correctly the first time the 3 numbers on a combination lock with 100 numbers possible in each place.

(4) Write down the combinations of a, b, c, d, e taken 3 at a time, and show that the number of those that do not contain a is ${}^4C_3$, while the number of those that contain a is ${}^4C_2$.

By generalizing this result prove that

$${}^nC_r = \overline{{}^{n-1}C_r} + \overline{{}^{n-1}C_{r-1}}.$$

Verify this by using the formulae for ${}^nC_r$ etc.

(5) Write down the combinations 2 at a time of a, b, c, d: from each one (such as ab) form the 3-at-a-time combinations got by adding one of the other letters (such as abc, abd), and show that the 3-at a-time combinations will by this process be obtained 3 times over.

Show that ${}^nC_r = \frac{n-r-1}{r} \cdot {}^nC_{r-1}$.

(6) In ${}^{10}C_6$ what is the number of combinations in which a particular thing occurs?

(7) In ${}^9C_3$ show that the number of combinations in which a particular thing occurs is one-third of the whole number.

Show that the same is true for ${}^{3m}C_m$.

(8) Find the number of changes that can be rung on a peal of 7 bells, each bell to be used once, and the last bell being a definite one.

(9) How many different signals can be sent with 5 flags displayed (i) 3 at a time, (ii) 4 at a time, (iii) 2 at a time, (iv) any number at a time?

(10) How many straight lines are obtained by joining in all possible ways 4 points of which no 3 are in the same straight line?

How many points of intersection are there of 4 straight lines, no 2 of which are parallel, and no 3 of which meet in a point?

(11) In a telegraphic code there are two signs, a dot and a dash. How many letters can be made with these signs 1, 2, 3, or 4 at a time?

(12) In how many ways can 5 ladies and 5 gentlemen sit at a round table, so that no two ladies sit together?

If the positions of the host and hostess are fixed, in how many ways can the party sit?

(13) There are 4 letters and 4 directed envelopes. In how many ways can the letters be put into the envelopes, so that each is in a wrong one?

(14) In how many ways can 20 books be arranged on a shelf, so that a particular pair of books shall (i) come together in a particular order, (ii) not come together?

Ex. 96. BINOMIAL THEOREM.

(Positive integral index.)

Write out the expansions of the following:

(1) $(a+x)^5$. (2) $(a+b)^7$. (3) $(a-b)^6$.

(4) $(2x-y)^4$. (5) $(x-2)^6+(x+2)^6$.

(6) $\left(x+\frac{3}{x}\right)^4-\left(x-\frac{3}{x}\right)^4$.

Write down and simplify the coefficient

(7) Of x^4 in $(1-x^2)^{12}$. (8) Of y^3 in $(x-2y)^9$.

(9) Of t^5 in $(2x+3ty)^7$. (10) Of x^9 in $\left(x^2-\frac{3}{x}\right)^6$.

(11) Of x^7 in $(a^2-2x^2)^5(1+x)$.

Write down and simplify

(12) The 5^{th} term of $\left(2x+\frac{b}{3}\right)^7$.

(13) The middle term of $\left(\frac{x}{2}+y\right)^{12}$.

(14) The middle term of $\left(\frac{3x}{4a}-\frac{4a}{3x}\right)^8$.

(15) The term independent of x in $\left(x^2-\frac{1}{2x}\right)^{18}$

(16) The expansion of $(x+\sqrt{2})^6+(x-\sqrt{2})^6$.

(17) Write down the coefficients of $(1+x)^6$. Using detached coefficients obtain by multiplication the expansions of $(1+x)^7$, $(1+x)^8$, $(1+x)^9$. Deduce the 'addition rule' for the binomial coefficients, viz. coefficient of x^r in $(1+x)^n$ = sum of coefficients of x^r and x^{r-1} in $(1+x)^{n-1}$.

(18) Use the addition rule given in question 17 to construct a table of binomial coefficients from those of $(1+x)^9$ to those of $(1+x)^{14}$.

(19) Give the proof that ${}^nC_r = {}^{\overline{n-1}}C_r + {}^{\overline{n-1}}C_{r-1}$

(i) from first principles;

(ii) from the expressions for ${}^nC_r$, etc;

(iii) from the identity $(1+x)^n=(1+x)^{n-1}(1+x)$.

(20) Show by means of the identity

$$(1+x)^{n+2}=(1+x)^n(1+2x+x^2)$$

that $${}^{n+2}C_r={}^nC_r+{}^{2n}C_{r-1}+{}^nC_{r-2}.$$

(21) By giving special values to x in the expansion of $(1+x)^n$ prove that

(i) $1+{}^nC_1+{}^nC_2+\ldots+{}^nC_{n-1}+1=2^n$;

(ii) $1-{}^nC_1+{}^nC_2-{}^nC_3+\ldots=0$;

(iii) $1-2\,.\,{}^nC_1+4\,.\,{}^nC_2-8\,.\,{}^nC_3+\ldots=(-1)^n$.

(22) Find the sum of the coefficients in the expansions of

(i) $(x-y)^{12}$; (ii) $(x+2y)^5$; (iii) $(3x-2y)^{15}$.

(23) In the expansion of $\left(1+\frac{1}{3}\right)^{20}$ write down the ratio of the $(n+1)^{\text{th}}$ term to the n^{th}, and show that it is less than 1 if $n>5$.

(24) Write out the expansion of $\left(1+\frac{1}{5}\right)^{18}$ until the greatest term is reached.

(25) Find the greatest term in the expansions of

(i) $(1+\cdot3)^{20}$; (ii) $(1+\cdot07)^{50}$.

Ex. 97.

Assuming the Binomial Theorem in the form

$$(1+x)^n=1+n\,.\,x+\frac{n(n-1)}{1\,.\,2}x^2+\ldots$$

to be true if $x<1$ for all values of n, write down and simplify the first four terms in the expansions of

(1) $(1+x)^{\frac{1}{2}}$. (2) $(1+x)^{\frac{3}{2}}$. (3) $(1-x)^{\frac{1}{2}}$.

(4) $(1+x)^{-1}$. (5) $(1-x)^{-1}$. (6) $\left(1-\frac{x}{2}\right)^{-4}$.

(7) $(1-2x)^{-\frac{3}{2}}$. (8) $(2+x)^{-2}$. (9) $(a-x)^{-\frac{1}{2}}$.

Write down and simplify

(10) The 7^{th} term of $(1-2x)^{\frac{4}{3}}$.

(11) The coefficient of x^7 in $(1-2x)^{\frac{3}{2}}$.

(12) The 6^{th} term of $(1-x)^{-7}$.

(13) The $(n+1)^{\text{th}}$ term of $(1-x)^{-\frac{1}{2}}$.

Write down and simplify

(14) The r^{th} term of $(1+x)^{\frac{p}{q}}$.

(15) The $(r+1)^{\text{th}}$ term of $(1-x)^{-p}$.

(16) The $(n+1)^{\text{th}}$ term of $(1-x)^{-\frac{p}{q}}$.

Find to three places of decimals:

(17) $\sqrt{101}$. (18) $\sqrt[3]{1003}$. (19) $\sqrt[4]{626}$.

(20) $\sqrt{99}$. (21) $1 \div \sqrt[3]{123}$. (22) $\sqrt[6]{63}$.

Ex. 98. MISCELLANEOUS ARTIFICES.

(In connection with Equations.)

Solve the following equations:

[Guess answer and check; only one answer is possible.]

(1) $\left.\begin{aligned} x-y&=0,\\ 7x+5y&=12.\end{aligned}\right\}$ (2) $\left.\begin{aligned} \frac{x}{7}+\frac{y}{3}&=2,\\ \frac{5x}{7}-\frac{4y}{3}&=1.\end{aligned}\right\}$

(3) $\left.\begin{aligned} \frac{1}{3}x-\frac{1}{2}y&=0,\\ 3x-2y&=0.\end{aligned}\right\}$ (4) $\left.\begin{aligned} \frac{x}{a}+\frac{y}{b}&=2,\\ \frac{3x}{a}+\frac{2y}{b}&=5.\end{aligned}\right\}$

(5) $\left.\begin{aligned} bx+cy&=ax,\\ cx+ay&=bx.\end{aligned}\right\}$ (6) $\left.\begin{aligned} x+y&=c+d,\\ 7x-5y&=7c-5d.\end{aligned}\right\}$

(7) $\left.\begin{aligned} \frac{x}{l}-\frac{y}{m}&=0,\\ \frac{x}{l}+\frac{y}{m}&=4.\end{aligned}\right\}$ (8) $\left.\begin{aligned} bx+cy&=2bc,\\ x-y&=c-b.\end{aligned}\right\}$

[Change variable, *i.e.* solve for something else than x, y.]

(9) $\left.\begin{aligned} 7(5x-2y)+8(7y-x)&=35,\\ 3(5x-2y)-8(7y-x)&=15.\end{aligned}\right\}$ (10) $\left.\begin{aligned} \frac{8}{x}+\frac{4}{y}&=2,\\ \frac{2}{y}-\frac{3}{x}&=3.\end{aligned}\right\}$

(11) $\left.\begin{aligned}\frac{2}{x-3}+\frac{1}{y-2}&=2,\\ \frac{4}{x-3}+\frac{1}{y-2}&=3.\end{aligned}\right\}$ (12) $\left.\begin{aligned}\frac{4}{x}+\frac{7}{y}&=29,\\ \frac{1}{x}+\frac{3}{y}&=11.\end{aligned}\right\}$

(13) $\left.\begin{aligned}\frac{15}{x}+\frac{7}{y-1}&=29,\\ \frac{9}{x}+\frac{15}{y-1}&=39.\end{aligned}\right\}$ (14) $\left.\begin{aligned}5\sqrt{x+8}+6y&=17,\\ 6\sqrt{x+8}+5y&=16.\end{aligned}\right\}$

(15) A man walks 20 miles on the level and 18 down hill in 9 hrs., and 16 on the level and $13\frac{1}{2}$ down hill in 7 hrs. What is his pace on the level, and down hill, per hour?

(16) A man walks 6 miles on the level and 10 down hill in $3\frac{1}{2}$ hrs., and back in 5 hrs. 20 mins.; but coming back he only goes $\frac{3}{5}$ of his down hill pace up hill, and $\frac{3}{4}$ pace on the level. What were his original rates on the level, and down hill?

[Simplify by division.]

(17) $\left.\begin{aligned}13x+26y&=39,\\ 5x-5y&=25.\end{aligned}\right\}$ (18) $\left.\begin{aligned}11x+66y&=132,\\ 34y-17x&=68.\end{aligned}\right\}$

[Add and subtract, and simplify each result by division.]

(19) $\left.\begin{aligned}134x+57y&=479,\\ 57x+134y&=94.\end{aligned}\right\}$ (20) $\left.\begin{aligned}20x+31y&=115,\\ 31x+20y&=38.\end{aligned}\right\}$

(21) $\left.\begin{aligned}(a+b)x+(a-b)y&=2(a^2+b^2),\\ (a-b)x+(a+b)y&=2(a^2-b^2).\end{aligned}\right\}$

(22) $\left.\begin{aligned}lx+my&=lm(l^2+m^2),\\ mx+ly&=2l^2m^2.\end{aligned}\right\}$

(23) A man buys 153 sheep and 167 pigs for £876. 10s. If he had bought 167 sheep and 153 pigs they would have cost him £883. 10s. How much did a pig, and a sheep cost?

(24) Of two steamers which go equally fast in still water, one goes for 67 hrs. down a river and 74 up, but the other goes for 74 hrs. down and 67 up. The former travels 1664 miles, and the latter 1720. Find the rate of the river and the rate of the steamers.

(25) Two steamers, A and B, ply on a river between two points. It is found at the end of a year that one has gone 31,745 miles, and the other 32,130 miles. A day's run is either 14 hrs. against stream or 7 hrs. with. If A has gone 188 of the former and 177 of the latter, and B vice versa, find the rate of the two steamers (supposed the same) in still water, and the rate of the river.

[Many equations can be reduced to Quadratics by a change of variable.]

(26) $x^4 - 20x^2 + 64 = 0.$ (27) $8x^6 + 215x^3 = 27.$

(28) $x^4 - x^2 = 156.$ (29) $x^3 - 9x^{\frac{3}{2}} + 8 = 0.$

(30) $3x + 2x^{\frac{1}{2}} - 1 = 0.$ (31) $x^{-3} + \frac{1}{x^{\frac{3}{2}}} = 2.$

(32) $3x^{\frac{2}{3}} - 4x^{\frac{1}{3}} = 55.$ (33) $x + \frac{1}{x} + \frac{1}{x + \frac{1}{x}} = 2\frac{9}{10}.$

[An important case is that of 'reciprocal' equations, so called because substituting for x its reciprocal $\frac{1}{x}$ leaves them unaltered. Rearrange and solve for $x + \frac{1}{x}$.]

(34) Clear of fractions the equation given in question 33.

Solve the equations:

(35) $x^4 + 4x^3 + 6x^2 + 4x + 1 = 0.$

(36) $6x^4 + 5x^3 - 38x^2 + 5x + 6 = 0.$

(37) $x^4 - 3x^3 + 4x^2 - 3x + 1 = 0.$

(38) $4x^4 - 4x^3 + 5x^2 - 4x + 4 = 0.$

Solve by finding $x - \frac{1}{x}$:

(39) $x^4 - 2x^3 - 5x^2 + 2x + 1 = 0.$

(40) $x^4 + 7x^3 - 2x^2 - 7x + 1 = 0.$

Remove factor $x \pm 1$ and solve:

(41) $2x^3 + 7x^2 + 7x + 2 = 0.$ (42) $2x^3 + 7x^2 - 7x - 2 = 0.$

(43) $x^5 - 4x^4 + 3x^3 + 3x^2 - 4x + 1 = 0.$

(44) $x^5 + 1 = 0.$ (45) $x^3 + ax^2 + ax + 1 = 0.$

(46) Solve $x^4 - 3ax^3 + 2(a^2+1)x^2 - 3ax + 1 = 0.$

[Simultaneous quadratics which are unaltered if the signs of both x and y are changed, give a quadratic for $y:x$. Put $y = mx$ and solve for m.

The graphical interpretation is important: see Misc. Graphs, Examples 28 to 30.]

(47) $\left.\begin{aligned} x^2 + 2xy &= 12, \\ xy - 2y^2 &= 1. \end{aligned}\right\}$ (48) $\left.\begin{aligned} 3x^2 - 2xy &= 8, \\ y(x+y) &= 3. \end{aligned}\right\}$

(49) $\left.\begin{aligned} x^2 + y^2 &= 25, \\ 3xy - 2x^2 &= 18. \end{aligned}\right\}$ (50) $\left.\begin{aligned} 6x^2 - 13xy + 6y^2 &= 0, \\ 5x^2 - 2y^2 &= 2. \end{aligned}\right\}$

(51) $\left.\begin{aligned} xy &= 1, \\ x^2 + 4y^2 &= 36. \end{aligned}\right\}$ (52) $\left.\begin{aligned} x^2 + y^2 &= 16, \\ x^2 + 4y^2 &= 36. \end{aligned}\right\}$

(53) $\left.\begin{aligned} xy &= 1, \\ x^2 - y^2 &= 4. \end{aligned}\right\}$ (54) $\left.\begin{aligned} 4xy &= 9, \\ x^2 - y^2 &= 20. \end{aligned}\right\}$

Solve the equations:

(55) $\left.\begin{aligned} x^3 - y^3 &= 124, \\ x - y &= 4. \end{aligned}\right\}$ (56) $\left.\begin{aligned} x^2y + xy^2 &= 308, \\ xy &= 28. \end{aligned}\right\}$

(57) $\left.\begin{aligned} \frac{x^2}{y} - \frac{y^2}{x} &= 8\tfrac{2}{3}, \\ \frac{x}{y} - \frac{y}{x} &= 2\tfrac{2}{3}. \end{aligned}\right\}$ (58) $\left.\begin{aligned} \frac{x^2}{y^2} + \frac{y^2}{x^2} &= 2\tfrac{81}{400}, \\ x^2 + y^2 &= 41. \end{aligned}\right\}$

[Simultaneous quadratics which are symmetrical in x and y may be solved by finding $x+y$ and xy.]

(59) If $x + y = u$, $xy = v$, show that (i) $x^2 + y^2 = u^2 - 2v$;
(ii) $x^3 + y^3 = u^3 - 3uv$.

Solve the equations:

(60) $\left.\begin{aligned} x^2 + y^2 &= 4(x+y) + 9, \\ x^2 - 5xy + y^2 &= x + y. \end{aligned}\right\}$ (61) $\left.\begin{aligned} x^2 - xy + y^2 &= 19, \\ xy - 3 &= x + y. \end{aligned}\right\}$

(62) $\left.\begin{aligned} x^3 + y^3 &= 13(x+y), \\ x^2 - 2xy + y^2 &= x + y - 6. \end{aligned}\right\}$ (63) $\left.\begin{aligned} x^2 + y^2 &= 37, \\ x^2 - 5xy + y^2 &= x + y. \end{aligned}\right\}$

[In the following, subtract 1 or some other whole number from each term, and simplify each side of the equation separately.]

(64) $\dfrac{x+8}{x+5}-\dfrac{x+10}{x+7}=\dfrac{x+7}{x+4}-\dfrac{x+9}{x+6}$.

(65) $\dfrac{2x+29}{2x+1}-\dfrac{x+15}{x+1}=\dfrac{2x-2}{2x-4}-\dfrac{x+6}{x+5}$.

(66) $\dfrac{5x-9}{x-2}-\dfrac{4x-3}{x-1}=\dfrac{6x+13}{x+2}-\dfrac{5x+16}{x+3}$.

[Surds must be removed by squaring. As this usually introduces other solutions, it is essential to test by substitution, which, if any, of the solutions obtained satisfy the original equation. It is understood that $\sqrt{\ }$ means 'positive root.']

(67) If $x=2$ then $x^2=4$: why is the converse not true?

(68) Show that squaring both sides of $\sqrt{x+5}=8$ is equivalent to multiplying both sides of $\sqrt{x+5}-8=0$ by a certain factor.

(69) Solve the equations: (i) $x+\sqrt{5x+10}=8$;
(ii) $x-\sqrt{5x+10}=8$.

Solve the equations:

(70) $\sqrt{x^2-7}=7-x$. (71) $2x-\sqrt{2x-1}=x+2$.

(72) $\sqrt{3x+10}-\sqrt{2x-1}=2$.

(73) $\sqrt{5x-1}+\sqrt{7x-6}=\sqrt{20x+25}$.

(74) $2(a+b-2x)=5\sqrt{(a-x)(b-x)}$.

(75) $-2b+\sqrt{a^2+x^2}=\dfrac{2a^2}{\sqrt{a^2+x^2}}$.

Ex. 99. MISCELLANEOUS GRAPHS.

(1) Draw the graphs corresponding to
(i) $y+x=5$; (ii) $x=2y-3$; (iii) $x=4$.

(2) Draw the graphs corresponding to
(i) $2x-y=5$;
(ii) $2x-y=7$;
and comment on the pair of simultaneous equations formed by taking (i) and (ii) together.

(3) Draw the curve $y=x^2$, and by considering the positions of the four lines $y=\pm px\pm q$ (where p, q are positive),

Show that $x^2=px+q$ has real roots, of opposite signs, with the larger one positive;

$x^2=-px+q$ has real roots, of opposite signs, with the larger one negative;

$x^2=px-q$ may not have real roots, but if it has both are positive;

$x^2=-px-q$ may not have real roots, but if it has both are negative.

Restate the above results in terms of a, b, c for the equation $ax^2+bx+c=0$.

[Any convenient number should be taken for q, and two or three different values of p used.]

(4) Find the roots of (i) $x^3=3x+2$; (ii) $x^3-5x+2=0$; (iii) $3x^3=x+1$.

(5) Draw the graph for $y=e^x$, where e is a constant, from the data:

$e^{-4}=\cdot02$, $e^{-3}=\cdot05$, $e^{-2}=\cdot14$, $e^{-1}=\cdot37$, $e^0=1$, $e^{\frac{1}{2}}=1\cdot65$, $e^1=2\cdot72$, $e^{\frac{3}{2}}=4\cdot48$, $e^2=7\cdot39$, $e^{\frac{5}{2}}=12\cdot2$, $e^3=20\cdot1$.

(6) The British ton $=1\cdot02$ times the French ton of 1000 kilogrammes. Draw a graph for converting from one scale to the other, and find (i) what decimal of a British ton is equal to one French ton; (ii) the number of kilogrammes in 7 cwt.

(7) Draw the graph of $y=\frac{1}{x}$.

[Between $x=\pm1$ take values of x differing by $\cdot1$, elsewhere take integral values of x.]

(8) Give the following table of corresponding values:

x	55	56	57	58	59	60
y	7·42	7·48	7·55	7·62	7·68	7·75

Estimate (i) the value of y for $x=57\cdot6$;

(ii) the value of x for $y=7\cdot5$.

(9) Given the following table of corresponding values:

x	21	22	23	24	25
y	·366	·384	·401	·419	·436

Find for what value of x, y is 400;
for what value of y, x is 24·5.

(10) If x is real, prove that $x^2 - 8x + 22$ is not less than 6.

(11) The following data connect the circumference (C inches) of an iron-wire rope, and the weight (B tons) that is just sufficient to break it.

C	4	4·5	5	5·5	6	6·5	7	7·5
B	15·75	21·0	24·8	30·0	36·2	42·7	48·3	55

Exhibit graphically.

(12) Draw the following curve (which to speak roughly indicates the shape of waves):

x	0	1	2	3	4	5	6
y	0	·26	·5	·71	·87	·97	1·0

where (i) if the sign of x is changed, the sign of y is changed but its numerical value is unaltered, and (ii) as x increases from 6 to 12, y decreases just as it increased, *e.g.* for $x = 7$, y is the same as for $x = 5$.

(13) Show graphically the increase of population of a town given:

Year, - - - - -	1894	1895	1896	1897	1898	1899	1900	1901
Population in thousands,	1·08	1·1	1·13	1·22	1·46	1·57	1·62	1·65

Infer the probable population for 1902.

(14) A steel rod sags owing to a weight being placed in the centre:

Weight (grammes), -	100	120	140	150	180	200
Sag (cents.), - -	1·13	1·27	1·39	1·49	1·76	1·88

Infer the sag for 160 grammes.

(15) An elastic string is stretched by a weight:

Weight in oz., -	5	6	7	8	9	10
Length in inches, -	6·2	6·4	6·6	6·8	7·0	7·2

If the extension is proportional to the tension, infer the unstretched length.

(16) Solve the equation $x^3 = x + 1$:

(i) by drawing $y = x^3 - x$;

(ii) by drawing $y = x^3$ and $y = x + 1$.

(17) Find the maximum value of y where $y = 7 + 6x - x^2$, and explain how the roots of $7 + 6x - x^2 = a$ for different values of a are related to the value of x which makes y maximum.

(18) Trace $y = \sqrt{1 - x^2}$ for such values of x as make y real.

(19) Trace the curves:

(i) $y = (x - 1)(x - 2)(x - 3)$;

(ii) $y^2 = (x - 1)(x - 2)(x - 3)$.

(20) Explain in general terms the difference in shape between $y_1 = x^3$ and $y_2 = x^5$.

(21) Solve the equation $x^3 = x^2 - 4x + 4$.

(22) Solve the equation $x^4 = 2x + 1$.

(23) Draw the graph of $y = \dfrac{x^2 - 8x + 15}{x}$, finding the turning values of y.

(24) Trace the curve $y = \dfrac{x^2 - 5x + 6}{x^2 - 5x + 4}$.

(25) Draw the graph of $y = \dfrac{x^2 - 3x - 3}{2x^2 + 2x + 1}$.

(26) Find the volume of a cone 8 inches high, the sectional areas at various heights being given by

Height in inches, - - -	0	2	4	6	8
Sectional area in square inches, -	54	30·4	13·5	3·4	0

(27) Find the volume of a tree trunk 18 feet high, given the following:

Height in feet, - - - -	0	3	6	9	12	15	18
Sectional area in square feet, -	3·08	2·46	1·94	1·44	1·10	·94	·76

(28) Show that for the following curves if a point (a, b) lies on the curve, then the point $(-a, -b)$ does so also.

Trace the curves roughly, and show that their points of intersection lie two by two on straight lines through the origin, so that for the four points of intersection of any two of the curves there are only two values of $y:x$ (compare with Misc. Artifices, Nos. 47-58):

$$x^2+y^2=16, \qquad x^2-y^2=4,$$
$$xy=1, \qquad x^2+4y^2=36.$$

(29) Solve graphically the equations $xy=1$,
$x^2-y^2=4$,
drawing the parts of the curves near where they meet, on a large scale.

(30) Use the same figure as in the last question to solve the equations $x^2+y^2=16$, $x^2-y^2=4$.

(31) If $f(x)=(x-2)(x-3)(x-4)$, write down $f(x+a)$, and show that the roots of $f(x+a)=0$ are less by a than the roots of $f(x)=0$.

Draw the graph of $y=f(x)$, and show how to obtain the graph of $y=f(x+a)$. [In particular show the graph of $y=f(x-1)$.]

How is the absolute term of $f(x)$ shown in the graph?

Discuss the changes in the absolute term of $f(x+a)$ as a increases from 0 up to 5.

(32) Generalise the last question by explaining, if ϕ is any known function

(i) how from the graph of $y=\phi(x)$ we can obtain the graph of $y=\phi(x+a)$;

(ii) what general form can be given for the absolute term of $\phi(x+a)$.

Ex. 100. MISCELLANEOUS LOGARITHMS.

Find the approximate values of

(1) $1836 \times (4{\cdot}2)^3$.

(2) $1/(19{\cdot}27)^9$.

(3) $172 \times \sqrt{836 \times 2412 \times (3{\cdot}6)^{-5}}$.

(4) $\sqrt[5]{{\cdot}000364}$.

(5) $218^3 \times \sqrt[4]{{\cdot}124} \times (1{\cdot}031)^{17}$.

(6) $\sqrt{2289 \times 343 \times 1092 \times 854}$.

(7) $42 \times \sqrt{({\cdot}0016)^7} \div \sqrt[3]{108}$.

(8) $\sqrt[3]{{\cdot}0125} \times \sqrt{31{\cdot}15} \div {\cdot}00081$.

(9) $\sqrt[3]{\left(\dfrac{294 \times 125}{42 \times 32}\right)^2}$.

(10) $403{\cdot}1 \times {\cdot}002317 \times 17 \div 18{\cdot}54$.

(11) $\left\{\dfrac{330 \times \dfrac{1}{49}}{\sqrt[3]{22 \times 70}}\right\}^4$.

(12) $\dfrac{2}{3}\sqrt[7]{981}$.

(13) $6{\cdot}283 \times \sqrt{\dfrac{161{\cdot}4 \times 15}{32 \times 37{\cdot}1 \times (2{\cdot}4)^2}}$.

(14) Solve the equation $\left(\dfrac{6}{11}\right)^x = \left(\dfrac{3}{4}\right)^2$.

(15) Find x and y from the equations

$$\left.\begin{aligned} 2^x \,.\, 3^y &= 1 \\ 3^{x+1} \,.\, 2^{y-1} &= 1 \end{aligned}\right\}.$$

(16) Find the value of $\dfrac{(181^2 - 73^2) \times 4956}{5 \times 3{\cdot}142 \times 8397}$.

(17) If $P \,.\, V^{1\frac{1}{6}} = \dfrac{69000}{144}$,

(α) given $P = 89{\cdot}86$ find V,

(β) given $V = 2{\cdot}041$ find P.

(18) If waves of ultra-violet light which act on a photographic plate follow each other at a distance of 2948×10^{-8} centimetres, and if the velocity of light is $300{\cdot}4 \times 10^8$ centimetres per second, find the number of waves which strike the plate in 3 millionths of a second.

(19) If $$p_1 \,.\, v_1^{1{\cdot}404} = p_2 \,.\, v_2^{1{\cdot}404},$$

where $p_1 = 15 \times 144$, $p_2 = 30 \times 144$, $v_1 = 3$,

find v_2 and prove that

$$\frac{1}{{\cdot}404}\{p_2 v_2 - p_1 v_1\} = 3539$$

(20) Find the value of $\left\{\dfrac{(\cdot 02)^3 \times \sqrt{506 \cdot 2}}{37 \cdot 5 \times \frac{3}{5}} \text{ of } \dfrac{1}{\cdot 08}\right\}^{\frac{3}{16}}$

(21) If $2^x = 7 \,.\, 2^{\frac{1}{x}}$, find x to within one per cent.

(22) If $(\cdot 04)^{3x} = \cdot 001$, find x.

Find the value of

(23) $\sqrt[3]{\left(\dfrac{294 \times 125}{43 \times 37}\right)^2}$.

(24) $(a^3 + 2a^2b + 1 \cdot 222)^{\frac{3}{2}} \div (a^2b^2)^5$, where $a = \cdot 504$, $b = \cdot 309$.

(25) $6 \cdot 283 \sqrt{\dfrac{369 \cdot 1 \times 15 \cdot 05}{32 \times 155 \cdot 1 \times (2 \cdot 4)^2}}$

(26) Find x from

$$\frac{60}{13 \cdot 2} = 6 \cdot 283 \sqrt{\frac{(x + 161 \cdot 4) \times 153}{32 \times 1006 \times (2 \cdot 4)^2}}$$

(27) Trace $y = x^{\frac{1}{x}}$ by using for x the values 1, 2, 2·6, 2·7, 2·8, 3, 4. Infer for what value of x, y is maximum.

(28) If s is the sum to n terms of a G.P. whose first term is a and common ratio r,

prove that $n = \dfrac{\log(a + \overline{r - 1}s) - \log a}{\log r}$.

If $a = 1$, $r = 2$, and s is to be as little as possible over a million, find n.

[Using the following 7-figure logarithms when necessary:

$\log 1 \cdot 02 = \cdot 0086002$, $\log 1 \cdot 03 = \cdot 0128372$, $\log 1 \cdot 04 = \cdot 0170333$, $\log 1 \cdot 05 = \cdot 0211893$, $\log 1 \cdot 0275 = \cdot 0117818$.]

Find

(29) The amount of £280 after 50 years, and after 200 years, at 3 % compound interest.

(30) The amount of £6000 after 100 years, and after 250 years, at $2\frac{3}{4}$ % compound interest.

(31) The true present value of £1000 due 15 years hence, reckoning compound interest at 2 %.

(32) After how many years a sum of money will have multiplied itself by 500, reckoning compound interest at 5 %.

(33) The true present value of one million pounds due 100 years hence, compound interest being reckoned at 4 %.

(34) The true present value of an annuity of £100 a year for 15 years, the first payment to be made 20 years hence, and compound interest being reckoned at 3 %.

(35) The true present value of a perpetual annuity of £100 a year, the first payment to be made 40 years hence, and compound interest being reckoned at $2\frac{3}{4}$ %.

Ex. 101. MISCELLANEOUS EXAMPLES.

(1) Find the value of

$$(x+a)(x+2a)(x+3a)-x(x+a)(x+2a),$$

when $x=-3a=6$.

(2) If $x=3$, $y=-1$, $z=11$, prove that

$$x^4-x^3y+x^2y^2-xy^3+y^4=z^2.$$

(3) Simplify $16x-17+\{8x-(9x-\overline{3-6x})\}$, and find the least integral value of x which will make the result positive.

(4) Prove that

$$(x-y)^3+(y-z)^3+(z-x)^3=3(x-y)(y-z)(z-x).$$

(5) If $a=y+z-2x$, $b=z+x-2y$, $c=x+y-2z$, find in terms of x, y, z the value of $a^2+b^2-c^2+2ab$.

(6) Solve the equations:

(i) $\frac{5x}{2}+\frac{9x}{4}=12+\frac{7x}{4}$;

(ii) $\frac{x+1}{2}+\frac{x}{3}=\frac{5-x}{4}+13\frac{1}{3}$.

(7) Verify the identity:

$$a^2(b-c)+b^2(c-a)+c^2(a-b)$$
$$=-a(b^2-c^2)-b(c^2-a^2)-c(a^2-b^2)$$

in the particular case when $a=10$, $b=9$, $c=8$.

(8) Multiply the square of the sum of the cubes of a and b by the cube of the sum of their squares.

(9) Prove that

$$(a+b-c)^3+(a-b+c)^3-8a^3=6a\{(b-c)^2-a^2\}.$$

(10) Solve the equation: $\cdot 03\,.\,x+1\cdot 07x=\cdot 1x+1\cdot 5$.

(11) The difference of the squares of two consecutive numbers is 19: find the numbers.

(12) If $a=1$, $b=2$, $c=3$, find the value of

$$\frac{3a^2-(2ab-5b^2-\overline{5c^2+2ac})+2bc}{4a^2+(2ab-\overline{ac-3b^2-2c^2})+bc}.$$

(13) Make a table showing the values of the two expressions x^2-3x and $3x^2-x$ for integral values of x from 0 to 10.

(14) Multiply $a^2-(b-c)^2$ by $b^2-(c-a)^2$.

(15) Simplify: (i) $\frac{x}{2}-\frac{x}{5}+\frac{x}{15}$;

(ii) $\frac{3x}{4}-\frac{5x}{7}-\frac{x}{28}$.

(16) Add together

$$\tfrac{1}{2}x^2+y^2-\tfrac{1}{3}z^2,\ \tfrac{1}{3}x^2-\tfrac{2}{3}y^2+z^2,\ x^2-\tfrac{1}{2}y^2-\tfrac{3}{4}z^2.$$

(17) Solve the equation: $\frac{x}{2}+\frac{2x-1}{2}=\frac{7x}{5}+1$.

If $x=9$, find the difference between the two sides of the equation.

(18) Prove that

$$(1+x)^2(1+y^2)-(1+x^2)(1+y)^2=2(x-y)(1-xy).$$

(19) Solve the equation: $\frac{5+3x}{2}-\frac{4x-7}{3}=\frac{16x-27}{21}-\frac{x+3}{5}$.

(20) Solve the equations: $\left.\begin{aligned}\tfrac{3}{4}x-2y&=1,\\ \tfrac{1}{3}x-y&=0.\end{aligned}\right\}$

(21) What number must be added to $x^3+x^2-4(x+3)$ that it may be divisible by $x-6$ without remainder?

(22) Solve the equations: $\left.\begin{aligned} \frac{y}{3}+\frac{2y-3x}{6} &= 8, \\ \frac{7x-3y}{2}-x &= 11. \end{aligned}\right\}$

(23) Divide $7x^5-5x^3+4x$ by x^2-6x+5.

(24) Simplify

$$(a+b+c)(a+b+d)+(a+c+d)(b+c+d)-(a+b+c+d)^2.$$

(25) Find the difference between the squares of $n+1$ and n, and use this result to calculate the squares of the whole numbers from 100 to 112.

(26) Simplify $\frac{3}{4}\frac{x^6}{y^4}\div\left(\frac{2}{3}\frac{x^7}{y^7}\right)\times\frac{2^4x}{3^3y}$.

Find also its value when x is replaced by $2x$.

(27) Solve the equation: $4-\frac{7-3x}{5}=3-\frac{3-7x}{10}+\frac{x+1}{2}$.

(28) Divide $8x^9-39x^7+66x^5-43x^3+8$ by $(x-1)^3$.

(29) Solve the equation: $\frac{2x-1}{2}+\frac{3x-2}{4}+\frac{5x-4}{8}=1-\frac{7x-6}{8}$.

(30) If P stands for x^5-4x^3+31x,

(i) from what must P be subtracted to leave remainder 0;

(ii) to what must P be added to give sum 0;

(iii) by what must P be multiplied that the product may be 0.

(31) If $x=\frac{pc-qb}{pa-rb}$, $y=\frac{rc-qa}{rb-pa}$, find the value of $ax+by-c$.

(32) Factorise: (i) $x^2+17x+60$; (ii) $x^2+17x-60$;
(iii) $x^2-17x+60$; (iv) $x^2-17x-60$.

(33) Divide $x^8+18x^7+38x^6+22x^5+1$ by $(x+1)^3$.

(34) A is 5 years more than twice as old as B. In 10 years time A will be 25 years older than B. How old are they?

(35) The same competitor has won an annual competition three times—twice running, and again after an interval. His age when he first won was two years more than three times the number of years between his first and third wins, but three years less than four times the number of years between his second and third wins. Find the age at which he first won.

(36) Factorise: (i) $4a^2 - 81$;

(ii) $5b^2 - 320$;

(iii) $x^2 + \frac{10}{x^2} - 7$.

(37) Solve the equations: $\frac{3}{x} + \frac{4}{y} = 1 = \frac{5}{x} - \frac{4}{y}$.

(38) If $A = a^2 + b^2 + c^2$ and $B = ab + bc + ca$, prove that

$$A^3 + 2B^3 - 3AB^2 = (a^3 + b^3 + c^3 - 3abc)^2.$$

(39) Find the L.C.M. of $x^3 - y^3$, $x^3 + y^3$, $x^4 + x^2y^2 + y^4$.

(40) Simplify $\frac{acx^2 + (ad - bc)x - bd}{a^2x^2 - b^2}$.

(41) Factorise: (i) $y^6 + z^6$;

(ii) $x^3 + x^2 - 14x - 24$.

(42) Solve the equation:

$$1{\cdot}2x - \frac{{\cdot}18x - {\cdot}025}{{\cdot}5} = {\cdot}4x + 4{\cdot}45.$$

(43) The incomes of A and B are in the ratio of 3 to 2, and their expenditures in the ratio of 5 to 3. Each saves £100 a year. Find their incomes.

(44) Divide $1 + 5x$ by $1 + 3x$ to five terms in ascending powers of x.

(45) If a certain war had lasted 37 days longer, its length would have been ten-ninths of what it would have been if the war had ended 63 days sooner. Find its length.

(46) Find the L.C.M. of $a^3 - b^3$, $a^3 + b^3$, $a^3 + a^2b$, $ab^2 + b^3$.

(47) Divide $a^2(b + c) + b^2(a - c) + c^2(a - b) + abc$ by $a + b + c$.

(48) Factorise: (i) $x^2 - 11xy + 30y^2$;

(ii) $(a - b)^2 - 7(a^2 - b^2) + 12(a + b)^2$;

(iii) $x^4 + 4x^2y^2 + 16y^4$.

(49) Prove that

$$(x - y)^4 + (y - z)^4 + (z - x)^4 = 2(x^2 + y^2 + z^2 - xy - yz - zx)^2 + 4xyz(x + y + z).$$

(50) Solve the equations:

(i) $\frac{1}{3}(5x+3)-\frac{1}{7}(16-5x)=37-4x$;

(ii) $\frac{3+10x}{5x-7}=9-\frac{7x+17}{x+1}$.

(51) If from £x. ys zd. be taken £z. ys. xd. leaving £a. bs. cd. as remainder, prove that £a. bs. cd. + £c. bs. ad. = a constant.

(52) Find the area of a right-angle isosceles triangle

(i) whose hypotenuse is a; (ii) whose perimeter is $2b$.

(53) Find the sides of a right-angled triangle given

(i) area, 216 sq. in.; hypotenuse, 30 in.;

(ii) hypotenuse, 25 in.; sum of other sides, 31 in.

(54) O, A, B are three points in order on a straight line. $OA=a$, $AB=\frac{9a}{2}$. Find where C must be taken in AB, so that if M be the middle point of BC, then $OA=\frac{1}{4}OM$.

(55) Simplify and put into factors

$$(x^3+a^3)^2+2ax(x^2-a^2)^2-a^4(x+a)^2.$$

(56) Simplify

(i) $\frac{2a^2+ab-b^2}{a^3+a^2b-a-b}$; (ii) $(a^2-b^2-c^2-2bc)\div\frac{a+b+c}{a-b-c}$.

(57) If one root of $x^2+px+q=0$ be double the other, prove that $2p^2=9q$.

(58) Solve the equation

$$\frac{x+a}{x-a}+\frac{x+b}{x-b}=2\left(\frac{a}{b}+\frac{b}{a}+1\right).$$

(59) If $x=\frac{4ab}{a+b}$, find the value of $\frac{x+2a}{x-2a}+\frac{x+2b}{x-2b}$ in terms of a and b.

(60) In how many ways can five boys sit on a form so that a certain couple shall not sit next each other.

(61) Show that the pair of equations

$$\left.\begin{aligned} y^2&=4ax,\\ y&=mx+\frac{a}{m},\end{aligned}\right\}$$

has equal roots whatever be the value of m.

(62) Show that the pair of equations

$$\left.\begin{aligned} x^2+y^2&=a^2,\\ y&=mx+a\sqrt{1+m^2},\end{aligned}\right\}$$

has equal roots whatever be the value of m.

[Examples 61, 62, are important in connection with Analytical Geometry.]

(63) If AB be divided internally at P so that $AP^2=AB.PB$, and externally at Q, so that $AQ^2=AB.QB$ prove that $PQ^2=5.AB^2$.

(64) Find the H.C.F. of

$$6x^3+x^2-11x-6 \text{ and } 6x^4-x^3-14x^2-x+6.$$

(65) If $\frac{a}{b}=\frac{c}{d}$, prove that $\dfrac{ab+b^2}{ac-ad}=\dfrac{ac+2bc+bd}{\dfrac{ac^2}{b}-cd}$.

(66) Find the 17th term and the sum to 17 terms of an A.P. whose 1st term is 49 and 2nd term 44.

(67) Simplify $\sum_{xyz}\dfrac{x(y+z-x)}{(x-y)(x-z)}$.

(68) If $\frac{x}{a}=\frac{y}{b}=\frac{z}{c}$, then $\dfrac{x^2-yz}{a^2-bc}=$ two similar expressions

$$=\left(\frac{x+y+z}{a+b+c}\right)^2.$$

(69) If $a=\dfrac{2}{2-b}$, $b=\dfrac{2}{2-c}$, $c=\dfrac{2}{2-d}$, $d=\dfrac{2}{2-x}$, prove that $a=x$.

(70) Simplify $\dfrac{3x^5-55x^2+8}{8x^5-55x^3+3}$.

(71) Simplify $\sum_{abc}\dfrac{1}{1+x^{a-b}+x^{a-c}}$.

(72) Simplify: (i) $\dfrac{1}{a+b}-\dfrac{1}{a-b}+\dfrac{2a}{a^2-b^2}$;

(ii) $1+\dfrac{2x+1}{2(x-1)}-\dfrac{4x+5}{2x+2}$.

(73) Solve the equations:

(i) $6x^2-17x+12=0$;

(ii) $(x-6)(x-5)+(x-7)(x-4)=10$.

(74) A and B buy cloth: B gave 7s. 6d. more for 60 yards than A did for 45 yards. B got 1 yard more than A did, for 7s. 6d. What did each pay?

(75) Divide $6x^6 - x^5 - 30x^4 + 17x^3 - 29x^2 + 5x + 4$ by $2x^3 - 5x^2 + 3x - 4$.

(76) Solve the equation $\frac{1}{x-1} - \frac{1}{x} = \frac{1}{x} - \frac{1}{x+2}$.

(77) Simplify $\frac{a^4 + a^2b^2 + b^4}{a^3 - b^3}$.

(78) Evaluate the continued product
$$(x^2 + x + 1)(x^2 + x - 1)(x^4 - 2x^3 + x^2 + 1).$$

(79) Insert three arithmetic means between ·012 and ·132.

(80) Solve the equations:
$$\left.\begin{aligned} 7x + 3y - z &= 10, \\ 8x - 2z &= 2, \\ 7x + y &= 9. \end{aligned}\right\}$$

(81) Simplify $\frac{\sqrt{5}-1}{\sqrt{5}-2} - \frac{\sqrt{5}-3}{\sqrt{5}+3}$

by rationalizing the denominators.

(82) Express $\sqrt{8 - 2\sqrt{15}}$ in form $\sqrt{a} - \sqrt{b}$.

(83) Solve the equations:

(i) $\frac{a-x}{b} - \frac{c}{a} = \frac{b-x}{a} - \frac{c}{b}$;

(ii) $\left.\begin{aligned} ax + by &= c^2, \\ bx - ay &= d^2; \end{aligned}\right\}$

(iii) $\left.\begin{aligned} (a+b)x - (a-b)y &= 3ab, \\ (a+b)y - (a-b)x &= ab. \end{aligned}\right\}$

(84) Simplify $\frac{b^2 - c^2}{ab - bc} \times \frac{ab + 2ac}{ab - 2bc} \times \frac{a^2 - 3ac + 2c^2}{b^2 + 3bc + 2c^2}$.

(85) If $\frac{a}{b} = \frac{c}{d}$, prove (i) $\frac{pa^2 + qb^2}{pc^2 + qd^2} = \frac{a^2 - b^2}{c^2 - d^2}$;

(ii) $\frac{2a^3c - (5a - 3b)abc}{ac^2(2c - 5d) + 3acd^2} = \frac{ab + b^2}{cd + d^2}$.

(86) Divide $4(a+b)x^2 + 2(a-c)xy - (b+c)y^2$ by $2x + y$.

(87) Solve the equation $\frac{7}{2x-3}+\frac{5}{x-1}=12$.

(88) If $\frac{a}{b}=\frac{c}{d}$, show that $\frac{ax+b}{cx+d}$ has the same value whatever x may be.

(89) If $x-\frac{1}{x}=1$, prove that $x^2+\frac{1}{x^2}=3$, and $x^3-\frac{1}{x^3}=4$.

(90) Solve the equation $\frac{a}{x-a}-\frac{b}{x-b}=\frac{a-b}{x-c}$.

(91) Solve the equations
$$\left.\begin{aligned}2x+3y&=12,\\3x+2z&=11,\\3y+4z&=10.\end{aligned}\right\}$$

(92) If $\frac{lx+ny+mz}{a}=\frac{my+lz+nx}{b}=\frac{nz+mx+ly}{}$ find $x:y:z$.

(93) Solve the equations
$$\left.\begin{aligned}xy-2x&=21,\\xy+3y&=50.\end{aligned}\right\}$$

(94) Solve the equations:

(i) $(x+5)^2-9(x+5)(2x+1)+20(2x+1)^2=0$,

(ii) $(3x-7)(2x-9)-(5x-12)(x-6)=(x-2)(2x-3)$.

(95) Solve the equations
$$\left.\begin{aligned}x^2+xy+x&=14,\\y^2+xy+y&=28.\end{aligned}\right\}$$

(96) Insert four arithmetic means between 2 and -18, and find the 50th term (counting from the 2) of the series obtained.

(97) Find the relations between p, q, r if x^3+px^2+qx+r is a perfect cube.

(98) Find the arithmetic progression whose 7th and 10th terms are respectively 15 and 21.

(99) Sum to $2n$ terms the series
$$a-2a+3a-4a+\ \ldots$$

(100) If a is not equal to b,
$$\text{and if } \frac{a^2(b-c)}{a-d}=\frac{b^2(a-c)}{b-d},$$
$$\text{prove that } \frac{1}{a}+\frac{1}{b}=\frac{1}{c}+\frac{1}{d}.$$

(101) Simplify $\frac{5}{3x-9}-\frac{8}{5x-15}$.

(102) Solve the equation $\frac{x+2}{x-3}+\frac{x-2}{x-6}=2$.

(103) Divide $a^8-b^8+a^2b^2(a^4-b^4)$
by $(a^2-ab+b^2)(a^2+ab+b^2)$.

(104) Simplify $\frac{x^2-x-6}{x^3+5x^2+8x+4}\times\frac{x^2-2x-8}{x^2-7x+12}$.

(105) If $x=a+b+\frac{(a-b)^2}{4(a+b)}$, $y=\frac{a+b}{4}+\frac{ab}{a+b}$,
prove that $(x-a)^2-(y-b)^2=b^2$.

(106) Solve the equations:

(i) $\frac{6x+4}{5}-\frac{15-2x}{x-3}=\frac{7(x-1)}{5}$,

(ii) $x^2+2xy-y^2=7(x-y)$,
$2x-y=5$.

(107) Which is the greater:
$\sqrt{10}+\sqrt{12}$ or $\sqrt{19}+\sqrt{3}$?

(108) Sum to infinity the series:

(i) $1+\frac{3}{4}+\frac{9}{16}+\dots$,

(ii) $a-\frac{b}{a}+\frac{b^2}{a^3}-\dots$.

(109) Solve the equations:

(i) $\frac{3(x-2a)}{b}+\frac{2(x-3b)}{a}=13$,

(ii) $\frac{a(2x+1)}{b}-\frac{5ax-4b}{5b}=\frac{4}{5}$.

(110) If $\frac{xy-ay-bx}{ax+by-xy}=\frac{y}{x}\times\frac{x-a+b}{a+b-y}$,
then either $x=y$ or $\frac{2}{a+b}=\frac{1}{x}+\frac{1}{y}$.

(111) If $(a+bc)^2(1-a^2)=(b+ac)^2(1-b^2)$,
prove that $a=\pm b$ or $a^2+b^2+c^2+2abc=1$.

(112) A's income is half B's: B spends £60 a year more than A. At the end of 2 years A has saved £200 and B has saved £600. Find their incomes.

(113) Solve the equations

$$\text{(i)}\quad 12x^2-31ax+20a^2=0.$$

$$\text{(ii)}\quad \left.\begin{aligned} x^2+xy&=140\\ y^2+xy&=56 \end{aligned}\right\}.$$

$$\text{(iii)}\quad \frac{1+x+x^2}{1+y+y^2}=3.$$
$$x+y=6.$$

(114) If A, B, C are any 3 points on a straight line, and P any other point on it, prove that

$$AB.CP^2+BC.AP^2+CA.BP^2=-AB.BC.CA,$$

regard being had to the signs of the segments.

(115) PQ is bisected at M and produced to A on one side and to B on the other, so that $AP>BQ$. If C is the middle point of AB, prove that $AP^2-BQ^2=2AB.CM-2PQ.CM$.

(116) Solve the equation

$$\frac{2x^2-x-1}{x-2}+\frac{2x^2-3x-1}{x-3}=\frac{8x^2-9}{2x-3}.$$

(117) If x varies as y, and xy varies as z^2, prove that $x^3+y^3+z^3$ varies as xyz.

(118) When is $(a^{n-1}-b^{n-1})(a^n-b^n)(a^{n+1}-b^{n+1})$ divisible by $(a-b)^3(a+b)^2$?

(119) If a, b, c are unequal, and if

$$\frac{(a+b)(c+d)}{ab+cd}=\frac{(a+c)(b+d)}{ac+bd},$$

then each $=\dfrac{(a+d)(b+c)}{ad+bc}$ and also $=-1$.

(120) Simplify $\dfrac{x-y}{x+y}+\dfrac{x+y}{x-y}$ if $x=2+\sqrt{3}$, $y=2-\sqrt{3}$.
and find its numerical value to 2 places of decimals.

(121) If D is the H.C.F. of A and B, prove that their L.C.M. is $\frac{AB}{D}$.

(122) Express $\sqrt{28+10\sqrt{3}}$ as $a+\sqrt{b}$.

(123) Prove that a rational integral function of x is divisible by $x+1$ if the sum of coefficients in even places = sum of coefficients in odd places; and is divisible by $x-1$ if the sum of the coefficients is 0.

(124) If all the quantities are real and if

$$(x-a)^2+(x-b)^2+(x-c)^2=0;$$

prove that $x=a=b=c$.

(125) Express in a single statement

(i) either $a=b$ or $c=d$;

(ii) $a=b$ and also $c=d$.

(126) Solve the equations

(i) $\left.\begin{aligned} 2x+3y&=12,\\ 3x+2z&=11,\\ 3y+4z&=10. \end{aligned}\right\}$

(ii) $x^2+xy+y^2=19=x^3-y^3$.

(iii) $\left.\begin{aligned} 2x^2-3xy+y^2&=3,\\ x^2+2xy-3y^2&=5. \end{aligned}\right\}$

(127) In an A.P. the first term is 5,
the last term is 23,
the sum is 392;
find the number of terms, and the common difference.

(128) If $ax+by=1$ prove that

$$ab(x^2+y^2)+(a^2+b^2)xy+(a-b)(x-y)=1.$$

(129) Sum to infinity $2\cdot5+\cdot5+\cdot1+\ldots$.

(130) Sum (i) 81, 79, 77 ... to 11 terms;
(ii) 4, 12, 20 ... to n terms.
Is 1889 a term of either of these series?

(131) Solve the equations

$$\text{(i)}\quad \frac{4x+1}{2x-1}-\frac{3x}{3x-2}+\frac{7x+5}{1-7x}=0;$$

$$\text{(ii)}\quad \frac{22a+b-3x}{14a-b-x}=\frac{6a+3b+x}{4a+b+x}.$$

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(132) A is 24, B 15 years old. How soon will the ratio of their ages be less than 7 : 5?

(133) Can x, y, z be found to satisfy the equations

$$\left.\begin{aligned} y+2x&=2z+1,\\ x+2z&=y+2,\\ x+2y&=4z+3. \end{aligned}\right\}$$

(134) If a, b, x are positive and $a>b$ prove that

$$\frac{a+x}{b+x}<\frac{a}{b}.$$

(135) If x, y, z are in A.P. prove that

$$x^2+xy+y^2,\quad x^2+xz+z^2,\quad y^2+yz+z^2$$

are also in A.P.

(136) Prove that the sum of $2n+1$ consecutive odd numbers beginning with $2n^2+1$ is $(n+1)^4-n^4$.

(137) Find in form $\sqrt{x}-\sqrt{y}$

$$\text{(i)}\quad \sqrt{312-2\sqrt{6647}}.$$

$$\text{(ii)}\quad \sqrt{a^2-a\sqrt{a^2-b^2}}.$$

(138) Solve the equation

$$2(x-1)(x-2)-\sqrt{x^2-3x+6}=20.$$

(139) Solve the equations

$$\text{(i)}\quad \left.\begin{aligned} 2(x^2+xy)&=-3y,\\ y^2+xy&=-6x. \end{aligned}\right\}$$

$$\text{(ii)}\quad \left.\begin{aligned} x^3+y^3&=351,\\ xy&=14. \end{aligned}\right\}$$

(140) If a is not equal to b and if $\dfrac{a+x}{b+x}=\sqrt{\dfrac{a}{b}}$ prove $x^2=ab$.

(141) Factorize $\sum_{abc} (x-a)^3(b-c)^3$.

(142) To 4 consecutive terms of an A.P. are added 5, 6, 9, 15 respectively. The result is 4 terms in G.P. Find what the A.P. was.

(143) Prove that $\dfrac{3\sqrt{5}-5\sqrt{3}+2\sqrt{30}}{\sqrt{5}+\sqrt{3}-2\sqrt{2}} = 3+2\sqrt{6}$.

(144) Sum to n terms

$$(x+a)+(x^2+2a)+(x^3+3a)+\ldots.$$

(145) Solve the equation

$$\frac{x+(x^2-4)^{\frac{1}{2}}}{x-(x^2-4)^{\frac{1}{2}}}+\frac{x-(x^2-4)^{\frac{1}{2}}}{x+(x^2-4)^{\frac{1}{2}}}=2.$$

(146) If

$$A=1-\left(p-\frac{3}{p}\right)x+x^2,$$

$$B=1+\left(p-\frac{3}{p}\right)x+x^2,$$

prove that $A^2+B^2+AB=3\left(x^2+\dfrac{p^2}{3}\right)\left(x^2+\dfrac{3}{p^2}\right)$.

(147) If a, b, c, d are in G.P., prove that

$$\frac{1}{a^2-b^2},\quad \frac{1}{b^2-c^2},\quad \frac{1}{c^2-d^2}$$ are also in G.P.

(148) Solve the equations

$$\text{(i)}\quad \left.\begin{aligned} 4x+3y&=17,\\ x^2+y^2&=13.\end{aligned}\right\}$$

$$\text{(ii)}\quad \left.\begin{aligned} \frac{x}{y}+\frac{y}{x}&=\frac{41}{20};\\ xy&=20.\end{aligned}\right\}$$

(149) If $\chi=x^2$, $\gamma=(x+1)^2$, $\zeta=2\{x^2+(x+1)^2+1\}$, prove that $\chi\gamma+\zeta$ and $\chi\zeta+\gamma$ are perfect squares.

(150) If α, β are the roots of $ax^2+bx+a+b=0$, prove that $(1-\alpha^2)(1-\beta^2)=4\alpha\beta$.

(151) If of $n+1$ terms in G.P. the first is a^n and the last b^n, prove that the sum is $\dfrac{a^{n+1}-b^{n+1}}{a-b}$.

(152) If $x^2+px+q=0$ has real roots show that

$$x^2+3px+2p^2+q=0 \text{ has real roots.}$$

(153) Of n straight lines in a plane no two are parallel;

Show by considering the total number of points of intersection that $(n-1)+(n-2)+\ldots+2+1=\frac{n(n-1)}{2}$.

(154) How many sides has a polygon which has 54 diagonals?

(155) Express as the product of four factors

$$\sum_{abc}(b^2-c^2)(1+ab)(1+ac).$$

(156) If $\log_a b=\log_b c=\log_c a$, prove $a=b=c$.

(157) £90 buys 51 tons of coal and 65 tons of coke; £3 buys 2 tons more coke than coal. Find the cost of each.

(158) Express $(a^2+b^2)(c^2+d^2)$ as the sum of two squares.

(159) If $x=\frac{a+b}{c-d}$ show that $(a-cx)^2+(x^2-1)(b^2-d^2)$ is a perfect square.

(160) If $\frac{xyz}{y+z}-x^2=\frac{xyz}{z+x}-y^2$, then each of these expressions $=\frac{xyz}{x+y}-z^2=yz+zx+xy$ unless $x=y$.

(161) Solve the equations

(i) $\frac{x+50}{x-25}+\frac{2x-50}{x+50}=3$;

(ii) $\frac{x-5}{x-7}+\frac{3(x-8)}{2x-15}=3$;

(iii) $\sqrt{a+x}+\sqrt{b+x}=2\sqrt{a+b+x}$.

(162) If $\beta=\left(\frac{1-e}{1+e}\right)^{\frac{1}{2}}$ prove that $\frac{1-\beta}{1+\beta}=\frac{e}{1+\sqrt{1-e^2}}$.

(163) In a G.P. whose first term is a and common ratio x, show that the n^{th} root of the product of the first n terms is $a.x^{\frac{n-1}{2}}$.

(164) Sum to infinity $(6-2\sqrt{3})+(4\sqrt{3}-6)+(9-5\sqrt{3})+\ldots$.

(165) Solve the equation $b^x = b^{x^2} \cdot a^{1-x}$.

(166) If
$$X = -pqx + (py + qz)(p + q),$$
$$Y = -pqy + (qx + pz)(p + q),$$
$$Z = -pqz + (px + qy)(p + q),$$
prove that $X^2 + Y^2 + Z^2 = (p^2 + pq + q^2)^2(x^2 + y^2 + z^2)$.

(167) Solve the equations

(i) $$\frac{x-5}{x-8} + \frac{x-1}{x-7} = \frac{4(x-3)}{2x-15};$$

(ii) $$\frac{cx-ab}{x+a} + \frac{ax-bc}{x+b} + \frac{bx-ac}{x+c} = 0,$$
where $a + b + c = 0$.

(168) If $x + h$ is the H.C.F. of $x^2 + ax + b$ and $x^2 + cx + d$, prove that their L.C.M. is
$$x^3 + (a + c - h)x^2 + (ac - h^2)x + h(a - h)(c - h).$$

(169) Sum to n terms and also to infinity
$$\frac{3}{10} + \frac{7}{10^2} + \frac{11}{10^3} + \frac{15}{10^4} + \ldots.$$

(170) Show that $(x + y)^7 - x^7 - y^7 = 7xy(x + y)(x^2 + xy + y^2)^2$.

(171) Sum to n terms, and to infinity, the series
$$4 + 7x + 10x^2 + 13x^3.$$

(172) A G.P. whose common ratio is $\frac{1}{2}$ has 2^n for its first term. Find the sum of all the terms after the n^{th}.

(173) If $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 0$ and $\frac{a}{x^2} + \frac{b}{y^2} + \frac{c}{z^2} = 0$, prove that
$$(b + c)x + (c + a)y + (a + b)z = 0.$$

(174) Given a semi-circle on AB as diameter, find a point P on it, such that if PM is drawn perpendicular to AB

(i) $AP = 2MP$,

(ii) $4MP^2 + AP^2 = k^2$,

(iii) $AM + PB = n$,

between what limits must the constants k, n lie that it may be possible to solve these problems. [$AB = a$, find AM.]

(175) If a, b are the first and last terms of an A.P., if a', b' are the p^{th} terms from the beginning and end respectively, if d is the common difference, and if n is the number of terms, prove that

$$a'b' - ab = (p-1)(n-p)d^2.$$

(176) A loan of £A bears interest at r per cent. In how many years will it be repaid by annual instalments of £R each, the instalments being applied first to pay the interest for the current year and secondly to reduce the principal.

(177) Prove that $\dfrac{7+3\sqrt{5}}{\sqrt{2}+\sqrt{7+3\sqrt{5}}} + \dfrac{7-3\sqrt{5}}{\sqrt{2}+\sqrt{7-3\sqrt{5}}} = 2\sqrt{2}.$

ANSWERS.

Ex. 1.

(4) 229, 85, 2205. (6) 22, 15, 24. (7) 32, 81, 98, 150, $\frac{1}{3}$, 12.

(8) $186=2.3.31$, $121=11^2$, $56=2^3.7$, $64=2^6$, $66=2.3.11$, $625=5^4$, $343=7^3$, $72=2^3.3^2$, $100=2^2.5^2$, $275=5^2.11$, $48=2^4.3$, $286=2.11.13$, $63=3^2.7$, $1715=5.7^3$, $41=41$, $85=5.17$, $120=2^3.3.5$, $105=3.5.7$.

(9) Either $\frac{1\,6\,4}{2}=82$ or $164=2.82$, etc. (11) 9, 0, 10, 9.

(12) x, 0, $x+1$, x. (13) 12, y. (14) 15, x.

Ex 2.

(7) 5°. (8) $2\frac{4}{9}°$.

Ex. 4.

(1) (i) 27; (ii) 64; (iii) 48. (2) (i) 13; (ii) 32; (iii) 0.

(3) (i) 35; (ii) −10; (iii) 18. (4) (i) 21; (ii) 14; (iii) 29.

(5) (i) 23; (ii) −24; (iii) 80; (iv) 0; (v) 147; (vi) 23.

(6) (i) 10; (ii) 32; (iii) 24; (iv) 132; (v) 0; (vi) 128.

(7) (i) 5; (ii) 7; (iii) −5; (iv) −23.

(8) (i) 0; (ii) 9; (iii) −12; (iv) −2.

(9) 2, 3, 18, 66, 83. (10) 0, 2, 92, 632, 882.

(11) −15, −18, 45, 150, 10182. (12) −6, −4, 360, 1820, 1009596.

(13) 7, 1, 1, 1, −5. (14) 24, 6, 0, 0, 0.

Ex. 5.

(13) (i) 4; (ii) 3; (iii) 1, 4; (iv) 3; (v) none; (vi) 1, 3, 4; (vii) 1.

(14) (i) −3; (ii) both; (iii) neither; (iv) −1.

(15) (i) 2; (ii) 5; (iii) 1, 2; (iv) 5, 2; (v) 1, 2, 5; (vi) none; (vii) 2.

(16) (i) 1; (ii) 0; (iii) none; (iv) 0, 1, −1.

Ex. 6.

(1) a. (2) $2b$. (3) $-4x$. (4) $-3y$.

(5) $3x^2$. (6) 0. (7) $2x+2y$. (8) $3c-ab$.

(9) x. (10) a^2+33a. (11) -9. (12) $3bx$.
(13) $4\frac{1}{2}x$. (14) $10a$. (15) $39abcd+2abc$.
(16) $-\frac{1}{2}b^2+4\frac{1}{2}b-4$. (17) $-12ab-2abc$. (18) $-1\frac{1}{2}x^2$.
(19) $cx+10x-2\frac{1}{2}c^2$. (20) $7\frac{1}{2}a-14b+8\frac{1}{2}c$.

Ex. 7.

(1) $4a-b$. (2) $5a$. (3) $2a+2c$.
(4) $5x-11y$. (5) $11a+2b+2c$. (6) $1+x^2$.
(7) $6x^2+2x+1$. (8) $3x^2-x$. (9) $3x^2+13x-3$.
(10) $6a^2x^2+14ax-5$. (11) $2+2x^2$. (12) $13x^2$.
(13) $19x^2-6x$. (14) $14-3x$. (15) $x+y+z$.
(16) $5b-3c$. (17) $4y^3+4y^2-1$. (18) $-x^2+16xy-4y^2$.
(19) $y+7$. (20) x^3+2x-6. (21) $7a$.
(22) x^3-2x^2+2x. (23) $x-3a$. (24) $9x^2$.
(25) $x+13y$. (26) $4x-y^2$. (27) 14. (28) 2.
(29) $x-b$. (30) $a-b$. (31) $2x$. (32) y.
(33) $-y$. (34) $4x$. (35) x^3-2x^2. (36) x^3+x-7.
(37) $3x-4$. (38) $2ab$. (39) $2a+2b$.
(40) $a^2-3a^2b+3ab^2-b^3$.

Ex. 8.

(1) $-a+10b+3c$. (2) $-a+8b-6c-2x$. (3) $5x+2$. (4) $2c$.
(5) $2a+3b-9b^2+bc+x$. (6) $-4x+6$. (7) x^2+x+6.
(8) $7x^2-4x+5$. (9) $2a-2b-2c$. (10) $x+3y$. (11) $5a$.
(12) $4a+7b$. (13) $2a$. (14) $2x$. (15) $-3x^3+17x^2+x-4$.

Ex. 11.

(1) x^5-x^3. (2) $-3bcx^7$. (3) $5x^3$. (4) $a+2b$.
(5) $56-21x^2$. (6) 0. (7) $2ax^3+6bx^3$. (8) $9b^2$. (9) $16x^2$.
(10) $-1331b^3c^3$. (11) b^4c^4. (12) a^5x^{10}. (13) $6a-3b$.
(14) $2ax^3-2axy^2$. (15) x^2-xy. (16) $x+y$. (17) $-bc$.
(18) $-\frac{1}{2}yz$. (19) $-3xy$. (20) $4a$. (21) $2a^2x^2y$. (22) $32x$.
(23) $2x^2$. (24) $6x^2$. (25) $12x^3y^4$. (26) $4x^2$. (27) $4x$.
(28) 27. (29) $60x$. (30) $3a$. (31) $2x+2$, $3x-3$, $\frac{x-3}{2}$.
(32) $2x-2$, $9x+6$, $6x+6$, $3x-9$, $8x$, $9y+18$.
(33) $x-8$, $7x-15$, $15y+5$.

Ex. 12.

(1) 0. (2) $45a - 14b + 5c - 2ac$. (3) 0.

(4) $a^2b - a^2c + b^2c - b^2a + c^2a - c^2b$. (5) $2bc + 2ca + 2ab$.

(6) $yz + zx + xy$. (7) $-ax + 4bx - 2ab$. (8) $2pt + qt - ps - ts$.

(9) 0. (10) $x^2 - y^2$. (11) $x^2 - y^2$. (12) $x^2 + 2x + 1$.

(13) 0. (14) 0. (15) 0. (16) $x^4 + y^4$.

(17) $x^2 - xy$. (18) $3x^3 + 3x^2y + xy^2 - y^3$.

(19) $ax - bx + by - cy + cz - az$.

(20) $a^2 + 2ab + b^2$, $a^2 - b^2$, $a^2 + ab + ac + bc$, $ac + ad + bc + bd$.

(21) $a^2 - 2ab + b^2$, $a^2 - ab - ac + bc$, $ac - bc + ad - bd$, $ac - bc - ad + bd$.

Ex. 14.

(1) 7. (2) 55. (3) 2. (4) $3\frac{1}{2}$. (5) -1.

(6) $2\frac{2}{3}$. (7) 5. (8) $\frac{1}{2}$. (9) -2. (10) $-1\frac{1}{2}$.

(11) $16\frac{5}{7}$. (12) 5. (13) 21. (14) $-1\frac{6}{25}$. (15) 4.

(16) -5. (17) 6. (18) $\frac{56}{81}$. (19) -21. (20) 4.

(21) 12. (22) 20. (23) 24. (24) 32. (25) 54.

(26) 1. (27) $-1\frac{3}{7}$. (28) $\frac{1}{2}$. (29) 32. (30) 12.

(31) 17. (32) 29. (33) 13. (34) 11. (35) 24.

(36) -6. (37) $\frac{1}{2}$. (38) $1\frac{2}{5}$. (39) $17\frac{2}{3}$. (40) $-\frac{13}{77}$.

(41) 2·3. (42) 1·7. (43) 3·1. (44) 2. (45) 3·28.

(46) $-16·4$. (47) $-1·26\ldots$. (48) $2·08\ldots$.

(49) $4·83\ldots$. (50) 18·63.

Ex. 16.

(1) 5. (2) 51. (3) 33. (4) 6. (5) $4\frac{1}{2}$.

(6) $3\frac{13}{27}$. (7) 34. (8) 116. (9) 31. (10) 5.

(11) 17. (12) $43\frac{1}{2}$. (13) 8. (14) 43. (15) 10.

(16) 7. (17) 10. (18) $14\frac{1}{2}$. (19) 66. (20) 348.

(21) $14\frac{2}{5}$. (22) 112. (23) 60. (24) 63, 70. (25) 51, 70.

(26) 59, 81. (27) $19\frac{1}{2}$, $20\frac{1}{2}$. (28) A £133, B £124.

(29) A £94, B £63. (30) A £24, B £34, C £42.

(31) A £114, B £57, C £62. (32) A £30, B £23, C £35.

(33) 45, 46. (34) 31, 33. (35) 38, 40, 42. (36) 20, 22.

(37) 21. (38) Father 30, son 3. (39) 23 years. (40) 6 years.

(41) 31. (42) A 48, B 32. (43) 66.

Ex. 18.

(1) $3\frac{1}{7}$, $6\frac{2}{7}$, $9\frac{3}{7}$, $12\frac{4}{7}$, $15\frac{5}{7}$. (2) $3\frac{1}{7}$, $12\frac{4}{7}$, $78\frac{4}{7}$, 154, $1257\frac{1}{7}$, 2464.

(3) 16, 64, 144, 256, 400, 576, 784, 1024, 1296, 1600; 3600, 4.

(4) 1200 lbs., 300 lbs. (5) 2925 lbs. (6) 2064 lbs.

(7) 171 lbs., $577\frac{1}{8}$ lbs., $50\frac{2}{3}$ lbs. (8) 168 lbs., 1344 lbs., 21 lbs.

(9) 2500. (10) (i) 7500; (ii) 870; (iii) 1150.

Ex. 19.

(1) $5a-7b+2c$. (3) 1, 5. (4) Yes.

(5) x^3+x^2. (6) (i) 4; (ii) 10. (7) $5x^2-x+3$.

(8) $2\frac{6}{7}$. (10) $3a^2+4ab+b^2$. (11) $\frac{xz}{y}$ pence, $\frac{ab}{12}$ pence.

(12) 50. (13) $3x^3+9ax-3a^2$. (14) $-2b-2c$.

(16) $a+b$. (17) (i) 99; (ii) 6·44. (18) 27, 28, 29.

(19) $-x^4+x^3+25$. (20) xy miles, $\frac{60a}{b}$ miles. (21) 154 sq. in.

(22) 42. (23) 9. (24) 16 years. (26) 3, -102.

(27) $4x^2-2x+29$. (28) $\frac{1}{2}x^2+\frac{1}{2}xy+\frac{1}{3}y^2$. (29) 8, 33.

(30) 44, 46. (31) (i) $3x^8y^4$; (ii) $-243a^5b^5$. (32) $\frac{13x}{6}+\frac{3y}{2}+\frac{z}{2}$.

(33) $-(2a+b+c)x^3$. (34) 27 ft., 18 ft. (35) $\frac{£xy}{40}$, 20

(36) $a^3+23a^2b+ab^2$. (37) $-5x^2-2xy-5xz$. (38) 48, 49.

(39) A £$244\frac{4}{9}$, B £$366\frac{2}{3}$, C £$488\frac{8}{9}$. (40) 54 ft.

Ex. 20.

(1) xy. (2) ab. (3) $2a$. (4) p^3q^3. (5) $4xyz$.

(6) $3a$. (7) $7a^2x$. (8) a^2b. (9) 5. (10) xy.

(11) $7bc^2$. (12) pr. (13) $9ax$. (14) $13ab$. (15) xyz.

Ex. 21.

(1) $3xyz^2$. (2) a^2b^3c. (3) $4x^3y^2$. (4) $12x^2y^3$.

(5) $9a^3b^3c$. (6) $5abx^3$. (7) xyz. (8) $l^2m^2n^2$.

(9) $a^2b^2c^2$. (10) $12a^2b^2c^2$. (11) $21a^2b^2c^2$. (12) $p^2q^2r^2$.

(13) $66a^3b^2$. (14) $30r^2s^3t^7$. (15) $102x^4y^2z^5$.

Ex. 22.

(1) x, $6x^2y^2z$. (2) c, $12abc$. (3) xy, $26ax^2y^2z$.

(4) $2abc$, $24a^2b^2c^2$. (5) 17, $170abcxyz$. (6) 1, $12ax^3y^2$.

(7) 1, $36\,p^2q^2r^2s^2$. (8) l, $4lm^2n^2$. (9) $7a$, $147a^2b^2c^2d$.

(10) x, $1155ax^3y^2z^2$.

Ex. 23.

(1) $\frac{a^3}{b^2}$. (2) $\frac{2c}{3b}$. (3) $\frac{a}{2}$. (4) $\frac{1}{4xyz}$. (5) $2a^3b$.

(6) $\frac{2a^4b^3}{xy}$. (7) $\frac{3a}{7b^2}$. (8) $\frac{1}{3ab}$. (9) $\frac{2ac^2}{3x^2}$. (10) $\frac{2}{5x}$.

(11) $\frac{2}{x}$. (12) $\frac{1}{l}$. (13) $-\frac{x^2}{3y}$. (14) $-\frac{x^5z}{2}$. (15) $\frac{8x^2y^3z^5}{9ab}$.

Ex. 24.

(1) $\frac{bx}{6a}$. (2) $\frac{3a}{2by^2}$. (3) $\frac{x}{pr}$. (4) $\frac{1}{c}$. (5) y.

(6) $\frac{3a^2}{bcnz}$. (7) 1. (8) $\frac{a}{z}$. (9) $\frac{9x}{2}$. (10) $\frac{x}{10}$.

(11) $-\frac{49b}{64xy}$. (12) $-mpq^2$. (13) $7x$. (14) $\frac{1}{y}$.

Ex. 26.

(1) $\frac{5x}{6}$. (2) $\frac{3a}{28}$. (3) $\frac{11b}{30}$. (4) $\frac{5y}{12}$.

(5) $\frac{7x}{12}$. (6) $\frac{10a-3b}{12}$. (7) $\frac{13x+11}{15}$. (8) $\frac{x+5}{8}$.

(9) $\frac{x}{12}$. (10) $\frac{5}{3x}$. (11) $\frac{a}{3b}$. (12) $\frac{13x^2}{4ab}$.

(13) $\frac{2a-3b}{bx}$. (14) $\frac{y}{x}$. (15) 0. (16) $\frac{a^3-y^3}{a^2}$.

(17) $\frac{n^3-2a^3}{2n^2}$. (18) $\frac{yz+zx+xy}{xyz}$. (19) 0. (20) $\frac{ax^2+2bcx-bc}{ax^2}$.

Ex. 27.

(1) a^4b^2. (2) $x^2y^2z^4$. (3) $4a^2b^4c^2$. (4) $9a^6$.

(5) $49a^{10}b^2$. (6) $144x^8z^2$. (7) $16a^2b^4c^2z^2$. (8) $9a^6x^{14}$.

(9) $25a^2x^4$. (10) $100a^{14}x^2y^2$. (11) $\frac{x^2}{4}$. (12) $\frac{4x^4y^2z^2}{9}$.

(13) $\frac{1}{25x^6}$. (14) $\frac{25x^2b^4}{4a^2y^2}$. (15) $\frac{36y^2z^2}{49a^4b^6}$. (16) $\frac{16a^6b^2c^2}{81x^6z^4}$.

(17) a^3b^6. (18) $8x^3y^3$. (19) $27a^3x^9$. (20) $27a^6y^9z^3$.

(21) $-x^3z^{12}$. (22) $-64l^3m^6$. (23) $125x^{21}$. (24) $-27y^{27}z^6$.

(25) $\frac{x^3}{8}$. (26) $-\frac{x^3y^3z^3}{27}$. (27) $\frac{8a^6b^3c^3}{l^3m^3}$. (28) $-\frac{64a^3b^9}{125x^{15}}$.

(29) $16a^4b^{12}$. (30) $81x^4y^8$. (31) $-a^{35}$. (32) $a^{24}b^4c^4$.

(33) $32a^5b^{10}$. (34) $-p^7q^{14}$. (35) $p^{16}q^{24}$. (36) $-\frac{8a^3b^9}{27x^3y^6}$.

(37) $\frac{x^5z^{10}}{y^5w^{15}}$. (38) $81xy^2$. (39) $xy^{10}z^{15}$. (40) $-p^9q^6$.

(41) $162x^5yz^5$. (42) $-a^7b^{23}$. (43) $-\frac{x^{10}}{y}$.

Ex. 28.

(1) a^3. (2) a^2b. (3) $a^4x^2y^2$. (4) xy^3z^2.

(5) $2ax^3$. (6) $3bc^3$. (7) $5cz^6$. (8) $a^{12}b^8c$.

(9) $4x^3y^{10}$. (10) $9y^{20}$. (11) $\frac{x^4}{4}$. (12) $\frac{2yz^4}{3}$.

(13) $\frac{13r^4}{5z}$. (14) $\frac{12b^3p}{14c^4}$. (15) $\frac{7l^2mn}{11pqr^2}$. (16) a^2.

(17) bx^3. (18) $2x^4$. (19) $4zw^5$. (20) $-b^4$.

(21) $-3c^5d^{14}$. (22) $-5xz$. (23) $\frac{2l}{3m^2}$. (24) $\frac{-4p^2r^5}{5x^{27}}$.

(25) $\frac{x^3}{7yz^2}$. (26) $\frac{-6y^4}{5z^3w}$. (27) cd^3. (28) x^2y^3.

(29) $2x^5$. (30) $-4lz^2$. (31) $-x^4y$. (32) x^3z^7.

(33) $\frac{xz^2}{2}$. (34) $\frac{2}{a^2b^3}$. (35) $-\frac{x^2}{y^3z}$.

Ex. 29.

(1) 5, 2. (2) 137, 73. (3) 16, 22. (4) 13, 20.

(5) $6\frac{1}{2}$, $2\frac{1}{2}$. (6) 7, 4. (7) 9, 1. (8) $3\frac{1}{8}$, $\frac{3}{4}$.

(9) 8, −13. (10) 1, 2. (11) 17, 1. (12) $\frac{1}{2}$, 4.

(13) 7, 2. (14) $-\frac{5}{7}$, $3\frac{5}{7}$. (15) 1, −1. (16) 2, 5.

(17) −4, −1. (18) $-\frac{1}{2}$, 1. (19) −5, −2. (20) $2\frac{12}{17}$, $-\frac{7}{51}$.

(21) 6934, 1088. (22) 8, 6. (23) 9, 7. (24) 20, $-\frac{1}{4}$.

(25) −7, 7. (26) 5, 2. (27) 4, 4. (28) $-2\frac{1}{2}$, $5\frac{1}{2}$.

(29) $\frac{1}{4}$, 1. (30) $\frac{1}{5}$, $\frac{2}{3}$. (31) 43, 33. (32) $1\frac{1}{2}$, $-\frac{1}{2}$.

(33) 12, 6. (34) 5, 5. (35) $\frac{1}{2}$, $\frac{1}{3}$. (36) $\frac{307}{331}$, $1\frac{70}{331}$.

(37) 30, $-7\frac{1}{2}$. (38) 1, 1. (39) $\frac{1}{2}$, $-1\frac{1}{2}$. (40) $\frac{1}{5}$, $\frac{1}{4}$.

(41) (*a*) $\frac{1}{2}$, $\frac{1}{2}$; (*b*) $-\frac{1}{4}$, $-\frac{1}{4}$; (*c*) $1\frac{1}{3}$, $1\frac{1}{3}$; (*d*) $1\frac{1}{2}$, $1\frac{1}{2}$; (*e*) 3, 3.

(42) (*a*) $\frac{1}{2}$, $\frac{1}{2}$; (*b*) −1, 0; (*c*) 2, 1; (*d*) $3\frac{1}{2}$, $1\frac{1}{2}$; (*e*) 3, $1\frac{1}{3}$.

(43) (*a*) $1\frac{1}{3}$, $-\frac{1}{3}$; (*b*) $\frac{4}{5}$, $-\frac{3}{5}$; (*c*) 3, $\frac{1}{2}$; (*d*) 5, $1\frac{1}{2}$; (*e*) 3, $\frac{1}{2}$.

(44) (*a*) $2\frac{1}{2}$, $-1\frac{1}{2}$; (*b*) $2\frac{3}{5}$, $-1\frac{1}{5}$; (*c*) $3\frac{1}{7}$, $\frac{3}{7}$; (*d*) $3\frac{1}{2}$, $1\frac{1}{2}$; (*e*) 3, 0.

(45) (*a*) −1, 2; (*b*) −1, 0; (*c*) −1, $2\frac{1}{2}$; (*d*) −1, $1\frac{1}{3}$; (*e*) impossible.

Ex. 30.

(1) 4, 3. (2) 36, 22. (3) 15, 3. (4) −8, 2.
(5) 8, 1. (6) $3\frac{1}{2}$, $-\frac{1}{2}$. (7) ·1, ·4. (8) 11, ·5.
(9) 2·43, 1. (10) ·7, 1·3. (11) 4·4, 1·43. (12) ·06, 4·2.
(13) 32, 48. (14) 14, 35. (15) 18, 4. (16) $-1\frac{2}{3}$, $3\frac{1}{2}$.
(17) 12, 9. (18) 10, 2. (19) 2, 1·3. (20) $\frac{19}{23}$, $\frac{22}{23}$.
(21) 6, −12. (22) −9, −26. (23) $-\frac{5}{8}$, $-\frac{1}{4}$. (24) $75\frac{1}{2}$, $\frac{25}{48}$.

Ex. 32.

(1) $\frac{1}{3}$, $1\frac{2}{3}$; $2\frac{1}{2}$, $-\frac{1}{2}$; −4, −7. (2) 1, 3; −2, 0; $-\frac{2}{3}$, −2.
(3) $4\frac{4}{5}$, $2\frac{2}{5}$; $10\frac{2}{7}$, $\frac{4}{7}$; −8, −4. (4) −3, −1; $2\frac{1}{2}$, $4\frac{1}{2}$; $2\frac{1}{2}$, $2\frac{2}{3}$.
(5) 1·62, 2·62 or − ·62, ·38. (6) 0, 0 or $\frac{2}{3}$, $\frac{4}{9}$.

Ex. 33.

(1) 2·7, 4·0. (2) 23·0, 27·8. (3) 1878, end of 1900.

Ex. 34.

(1) 2, 1, 4. (2) 3, 4, 1. (3) 4, 5, 4.
(4) 10, 1, 3. (5) −3, 2, 12. (6) 8, −6, 4.
(7) $4\frac{1}{2}$, $\frac{1}{2}$, $2\frac{1}{2}$. (8) $8\frac{1}{2}$, $5\frac{1}{2}$, $\frac{1}{2}$. (9) $5\frac{1}{2}$, $-2\frac{1}{2}$, $5\frac{1}{2}$, $1\frac{1}{2}$.
(10) −1, 2, 5. (11) $6\frac{1}{2}$, −5, $-2\frac{1}{2}$, 13. (12) 2, 1, 3.
(13) 1, 10, 3, 7, −2. (14) $2\frac{1}{2}$, −3, −1. (15) −6, 12, 2.
(16) 2, $4\frac{1}{2}$, $3\frac{1}{3}$. (17) 9, 21, −4. (18) 20, −7, 66.
(19) 12, $4\frac{2}{3}$, $-17\frac{1}{3}$. (20) $4\frac{22}{41}$, $11\frac{29}{41}$, $11\frac{31}{41}$.
(21) 7, 5, 8. (22) 36, 60, 40.

Ex. 35.

(1) 723, 661. (2) 478, 157. (3) 74, 50.
(4) 19, 9. (5) 555, 333. (6) 7, 5.
(7) 54, 48. (8) $5\frac{1}{2}$, $1\frac{1}{2}$. (9) 60, 12.
(10) 27, 5. (11) *A* £382, *B* £450. (12) *A* £$101\frac{1}{2}$, *B* £$98\frac{1}{2}$.
(13) *A* £$238\frac{1}{3}$, *B* £$136\frac{2}{3}$. (14) *A* £160, *B* £80, *C* £210, *D* £30.
(15) *B* £120, *C* £200, *D* £380. (16) Horse £25, cow £14.
(17) Cow £20, sheep £5. (18) 7 tandems, 19 singles.
(19) Father 52, son 14. (20) 16. (21) *A* 20, *B* 16.
(22) *A* 48, *B* 24. (23) $\frac{3}{4}$. (24) $\frac{7}{23}$.
(25) 963. (26) 200, 120, 40. (27) 225, 90, 45.
(28) 38, 27, 19. (29) 5, 4, 3, 2. (30) 26, 30, 14, 56.

Ex. 36.

(1) $x^2+7x+10$. (2) a^2+2a-3. (3) $y^2+3y-18$.

(4) a^2-4. (5) $b^2+3b-28$. (6) $-a^2+13a-40$.

(7) $ab-a+3b-3$. (8) $ax+4x-2a-8$. (9) $-x^2-x+12$.

(10) a^2-b^2. (11) $x^2+xy-2y^2$. (12) $2x^2-2c^2$.

(13) $x^2y^2-2xy-15$. (14) $a^2b^2-36x^2$. (15) $2x^2-3ax-5a^2$.

(16) $2x^2+xy-y^2$. (17) $2ax+8bx-ab-4b^2$. (18) x^4-y^4.

(19) $2a^2x^2-abx-b^2$. (20) $12-ab-a^2b^2$. (21) $3a^2x^2y^2-ax^2y-2x^4$.

The numbers given are the results of substituting in the two expressions and their product.

(22) $3x^3+7x^2+3x+2$; 5, 3, 15. (23) x^4-1; 15, 1, 15.

(24) $6a^4+a^3-8a^2+5a-1$; 3, 1, 3. (25) $4b^3-7b^2-3b+2$; 4, -1, -4.

(26) $21x^8-15x^7+2x^6+2x^5-9x^4+x^3-x^2+1$; 1, 2, 2.

(27) $a^4+a^2b^2+b^4$; 3, 1, 3. (28) $3a^5+5a^4+11a^3+4a^2-2a-6$; 3, 5, 15.

(29) y^4-1; 5, 3, 15. (30) $x^4+x^3y-xy^3-y^4$; 7, 3, 21.

(31) $3x^5+5x^4+4x^3+4x^2+x-1$; 4, 4, 16.

(32) 159951. (33) 239786. (34) 287742. (35) 1349431.

(36) 6, 15, 38, 52, 32; 79352. (37) 1, 10, 21, 45, 34, 5, 4; 2258454.

(38) $a^{10}-25a^6+20a^5-4a^4-10a^3+4a^2-1$; -3, 5, -15.

(39) $a^3+b^3-c^3+3abc$; 7, 2, 14. (40) $8x^3+27y^3+z^3-18xyz$; 3, 6, 18.

(41) $x^2+y^2+z^2+2yz+2zx+2xy$. (42) x^3-x^2-x+1.

(43) $a^3+3a^2b+3ab^2+b^3$. (44) $2a^3-3a^2b-3ab^2+2b^3$.

(45) $x^2+4y^2+9z^2+12yz+6zx+4xy$.

(46) Of $(a+b)^{10}$ coefs. are 1, 10, 45, 120, 210, 252, 210, 120, 45, 10, 1.

(47) Of $(a-b)^5$ coefs. are 1, -5, 10, -10, 5, -1.

(48) Coefs. are 1, 2, -3, -8, 2, 12, 2, -8, -3, 2, 1.

(49) Coefs. are 1, 4, 10, 16, 19, 16, 10, 4, 1.

(50) Coefs. are 1, 3, 6, 10, 12, 12, 10, 6, 3, 1.

Ex. 37.

(11) 4·8 feet. (12) If $l>m$, $XY=AB\left(\frac{m}{l-m}+\frac{m}{l+m}\right)=AB\,.\,\frac{2lm}{l^2-m^2}$.

Ex. 38.

(1) $4x^5-3x^3+6x$. (2) $2x^2+6xy+7y^2$. (3) $x^3-3x^2y+4xy^2$.

(4) $7x^2-33xy+14y^2$. (5) $a^2b+b^2c+c^2d+ad^2$. (6) $x+2$.

(7) $x-2$. (8) $x+2$. (9) $x+2$, remr. 1. (10) $x+1$, remr. 1.

(11) $x+4$. **(12)** $x-4\frac{1}{2}$, remr. $-\frac{1}{2}$. **(13)** $x+5$. **(14)** $2x+9$.

(15) (α) $x^3-6x^2+11x-6$; (β) x^3-4x^2+x+6;
(γ) x^3-3x^2-x+3; (δ) x^3-2x^2-x+2.

(16) (α) $x^3-3x^2-4x+12$; (β) x^3-x^2-8x, remr. 12;
(γ) x^3-7x-6; (δ) x^3+x^2-4x-4.

(17) (α) x^3-x^2+x-1; (β) x^3+x^2+x+1;
(γ) x^3+2x^2+4x+8, remr. 15; (δ) $x^3+3x^2+9x+27$, remr. 80.

(18) (α) $x^5-x^4-13x^3+13x^2+36x-36$. ($\beta$) $x^5+x^4-13x^3-13x^2+36x+36$.
(γ) $x^5+2x^4-10x^3-20x^2+9x+18$. ($\delta$) $x^5+3x^4-5x^3-15x^2+4x+12$.

(19) Coefs. all 1. **(20)** Coefs. +1 and −1 alternately.

Ex. 39.

(1) $x-1$. **(2)** $x+2$, remr. x. **(3)** x^2-4.

(4) (α) $3x-7$, remr. $24x-56$. (β) $\frac{3}{2}x-\frac{23}{4}$, remr. $\frac{147}{4}x-5$.
(γ) $\frac{x}{2}+\frac{5}{4}$, remr. $\frac{33x}{4}-\frac{77}{4}$ (δ) x, remr. $-x-28$.

(5) (α) $6x^2-11x-7$. (β) $3x^2-10x+7$.
(γ) x^2+3x-4. (δ) $2x^2+x-\frac{50}{3}$, remr. $\frac{193x}{3}-\frac{316}{3}$.

(6) (α) $6x-5$, remr. $-12x-48$. (β) $3x-7$.
(γ) $x+4$. (δ) $2x+3$, remr. $-41x-4$.

(7) (α) $6x^3-5x^2-18x-7$. (β) $3x^3-7x^2-3x+7$.
(γ) x^3+4x^2-x-4. (δ) $2x^3+3x^2-\frac{47x}{3}+\frac{43}{9}$, remr. $\frac{1501}{9}x+\frac{596}{9}$.

(8) $x^2+y^2+z^2+yz-zx+xy$. **(9)** $a^2+b^2+c^2-bc-ca-ab$.

(10) x^2-xy+y^2. **(11)** $3x+2y$, remr. $-16y^3$.

(12) $x^4-3x^3a+2x^2a^2+5xa^3-a^4$. **(13)** $a^3-3a^2b+2ab^2-1$.

(14) $1+x+x^2+x^3$. **(15)** $x+(p+a)$, remr. a^2+ap+q.

(16) $x+(a_1+t)$, remr. $t^2+a_1t+a_2$.

(17) $x^4+\overline{y+a_1}|x^3+\overline{y^2+a_1y+a_2}|x^2+\overline{y^3+a_1y^2+a_2y+a_3}|x$
$+y^4+a_1y^3+a_2y^2+a_3y+a_4$, remr. $y^5+a_1y^4+a_2y^3+a_3y^2+a_4y+a_5$.

(18) (i) x, remr. y^2; (ii) y, remr. x^2. **(19)** $x+y$.

(20) (i) $x^3+5x^2y+17xy^2+50y^3$, remr. $132xy^4-49y^5$;
(ii) $y^3+2y^2x+9yx^2+28x^3$, remr. $77yx^4-27x^5$.

(21) (i) $x^6+x^5y-x^3y^3-x^2y^4+y^6$, remr. xy^7-2y^8;
(ii) $-y^6-y^5x+y^3x^3+y^2x^4-x^6$, remr. $-yx^7+2x^8$.

(22) (i) $x^2-4xy-9y^2$, remr. $-31x^2y^3+6xy^4-10y^5$;
(ii) y^2+6x^2, remr. $-3y^2x^3+11yx^4-5x^5$.

Ex. 40.

(1) $x^4+7x^3+9x^2-4x+5$. (2) $x-7$. (3) a^8b^{16}, $-32a^5b^{15}$, $7x^2y^6$.

(4) (i) 5, 7; (ii) 14, 9. (5) cow £6, sheep £2. (6) x^4+x^3+4x-1.

(7) $a^3+b^3+c^3-3abc$. (8) $9abc$, $540a^4b^2c^3$.

(9) $x=2n+1$; $y-1$, y, $y+1$; $3y$. (10) 12, 17, 19. (11) x^2-x+2.

(12) $4yz$. (13) $2x^2+2$. (14) $x^7-x^6-x^5-9x^4-x^3-12x^2-x-6$.

(15) (i) 23; (ii) 52, 156. (16) (i) $-p^7q^7$; (ii) p^9.

(17) (i) $x^2-4xy+7y^2$, remr. $-8y^2$; (ii) $-y^2+4yx-7x^2$, remr. $8x^2$.

(18) $a^3+b^3+c^3+24abc$. (19) y is positive if $x>3\frac{1}{2}$. (20) $\frac{3}{4}$, $\frac{1}{4}$.

(23) The exp. is 0 if $x=1{\cdot}25$. (24) $p=5$, $q=-\frac{7}{5}$.

(25) (i) x^3-y^3; (ii) $x^2z-x^2y+y^2x-y^2z+z^2y-z^2x$; (iii) x^6-y^6.

(26) $2n-3$, $2n-1$, $2n+1$; $6n-3$. (27) $a+b-2x$. (28) 18 lbs.

(30) (i) ab, a^4b^4; (ii) 9, $108x^2y^3$; (iii) $13zw$, $546z^2w^2$.

(31) 50. (32) 44. (33) $74\frac{2}{3}$. (34) $62\frac{1}{2}$. (35) -6.

(36) $583\frac{1}{3}$. (37) $2\frac{1}{2}$. (38) $3\frac{1}{2}$ ft. (39) 20, 12. (40) 15.

Ex. 42.

(1) 5. (2) 28. (3) 1. (4) 4. (5) 0.

(6) 1. (7) x^5, x. (8) x^{10}, $-4x$. (9) x^{12}, 2. (10) $8x^5$, $15x$.

(11) $x^5+2x^4+2x^3+2x^2+x$. (12) $a^5+2a^4+a^3+a^2+2a+1$.

(13) y^4-4y^2+8y-4. (14) $2x^4+x^3+2x+1$

(15) $3x^5-11x^4+13x^3+7x^2-29x+15$. (16) $x^{16}+x^8+1$.

Ex. 43.

(1) $a(x+a)$. (2) $a(x-2a)$. (3) $3x(x-2)$.

(4) $4x(2x+1)$. (5) $y(x+2)$. (6) $x(x+1)$.

(7) $a^2(a-1)$. (8) $x^2(3x-8y)$. (9) $yz(2x+yz)$.

(10) $b^2(ac-6)$. (11) $p^2(p-3)$. (12) $lm(lm+2n^2)$.

(13) $b^2(b^2-3k^2)$. (14) $13a(a-3b)$. (15) $17(3x-2)$.

(16) $ax(x+2y-a)$. (17) $ad(7ad+28bc+6d^2)$.

(18) $7(x^2+2x-5)$. (19) $abc(a+b+c+d)$.

(20) $x^2y^2(xy+3xz-z^2)$. (21) $xy(34x^2y+2y-17x+6)$.

Ex. 44.

(1) $(x+a)(x+b)$. (2) $(x-c)(x+a)$. (3) $(2x+a)(x+b)$.

(4) $(ab-d)(ab+c)$. (5) $(2y+a)(y-b)$. (6) $(x-2c)(x+a)$.

(7) $(x-a)(x+7)$. (8) $(x-3)(x-c)$. (9) $(5+a)(2-b)$.

(10) $(3+b)(1-ax)$. (11) $(x+1)(a+b)$. (12) $(x^2-a^2)(p-q)$.

(13) $(x^2+a)(x+1)$. (14) $(x+1)(x^2+1)$. (15) $(x^2+1)(x-2)$.
(16) $(a+b)(a^2+b^2)$. (17) $(l-p)(n-l)$. (18) $(a^2-2)(a-3)$.
(19) $(t-z)(w+k)$. (20) $(x+y)(a^2b^2-3axz^2)$. (21) $(t-a)(b-a)$.
(22) $(a+b)(2x+y+z)$. (23) $(x+1)(x^4+x^2-1)$.
(24) $(x-z)(2a+b-c)$. (25) $(ax+1)(ay+2x+3)$.

Ex. 45.

(1) $(x+5)(x+3)$. (2) $(x+6)(x+1)$. (3) $(a+3)(a+2)$.
(4) $(x-6)(x-1)$. (5) $(b-2)(b-1)$. (6) $(x+7)(x+5)$.
(7) $(y-10)(y-10)$. (8) $(x+4)(x+7)$. (9) $(x-5)(x-8)$.
(10) $(x+9)(x+11)$. (11) $(x+3)^2$. (12) $(a-7)^2$.
(13) $(x-11)(x-12)$. (14) $(x+12)(x+13)$. (15) $(a-5b)(a-3b)$.
(16) $(ab+5)(ab+3)$. (17) $(l-7p)(l-9p)$. (18) $(xy+6z)(xy+z)$.
(19) $(4-x)(2-x)$. (20) $(11-y)(6-y)$. (21) $(3+p)(15+p)$.
(22) $(x+4)(x-3)$. (23) $(x+9)(x-8)$. (24) $(x-12)(x+7)$.
(25) $(x+6)(x-3)$. (26) $(a+11)(a-7)$. (27) $(b-9)(b+1)$.
(28) $(x-12)(x+2)$. (29) $(c-18)(c-2)$. (30) $(b-13)(b+6)$.
(31) $(x-1)^2$. (32) $(x-3)(x+1)$. (33) $(x-12)(x+10)$.
(34) $(3-x)(2+x)$. (35) $(7-x)(9+x)$. (36) $(1-x)(20+x)$.
(37) $(1-x)(20-x)$. (38) $(ab-5c)(ab+c)$. (39) $(x-yz)^2$.
(40) $(x-24)(x+3)$. (41) $(4-y)(16+y)$. (42) $(9+ab)(16-ab)$.
(43) $(a+27y)(a-9y)$. (44) $(c-3z)(c+27z)$. (45) $(x+2y)(x-y)$.
(46) $(x+13)(x-12)$. (47) $(x+71)(x-70)$. (48) $(ac+7)(ac+15)$.
(49) $(ac-30)(ac+8)$. (50) $(ac+33)(ac-11)$.

Ex. 46.

(1) $(a-x)^2$. (2) $(b+2)^2$. (3) $(x-2a)^2$.
(4) $(4x-a)^2$. (5) $(ab+c)^2$. (6) Not a square.
(7) $(7a+2b)^2$. (8) Not a square. (9) $(x-9y)^2$.
(10) $(2l-m)^2$. (11) $(abc+3z)^2$. (12) Not a square.
(13) $(2c-5d)^2$. (14) Not a square. (15) $(15abx-1)^2$.
(16) $\left(a+\frac{1}{x}\right)^2$. (17) $\left(\frac{a}{b}-x\right)^2$. (18) $\left(\frac{x}{2}-\frac{1}{y}\right)^2$.
(19) 4, $x+2$. (20) $4a^2$, $x-2a$. (21) $81b^2$, $x+9b$.
(22) 36, $y-6$. (23) 16, $x+4$. (24) 9, $a-3$.
(25) $36a^2$, $b-6a$. (26) $\frac{25}{4}$, $x+\frac{5}{2}$. (27) $\frac{9}{4}$, $y-\frac{3}{2}$.

(28) $\frac{1}{4}, x+\frac{1}{2}$. (29) $1, 2x+1$. (30) $9, 3y-3$.

(31) $9b^2, 4a-3b$. (32) $\frac{9c^2}{4}, ab+\frac{3c}{2}$. (33) $\frac{1}{4}, lm-\frac{1}{2}$.

(34) $\frac{1}{4}, 2y+\frac{1}{2}$. (35) $\frac{x^2}{196}, 7ab+\frac{x}{14}$. (36) $\frac{49a^2b^2}{4}, 3xyz-\frac{7ab}{2}$.

Ex. 47.

(1) $(a+3)(a-3)$. (2) $(b+2)(b-2)$. (3) $(cx+9)(cx-9)$.

(4) $(7+x)(7-x)$. (5) $(8+ax)(8-ax)$. (6) $(9-2a^3x^3)(9+2a^3x^3)$.

(7) $(20+x)(20-x)$. (8) $(x+3ay)(x-3ay)$. (9) $(10a+7)(10a-7)$.

(10) $(9y+bc)(9y-bc)$. (11) $(11+lm)(11-lm)$.

(12) $(13+2y)(13-2y)$. (13) $(x^2+3y^2)(x^2-3y^2)$.

(14) $(4a^2+5b^2)(4a^2-5b^2)$. (15) $(8a^3+c^3)(8a^3-c^3)$.

(16) $(15-z^5)(15+z^5)$. (17) $(11ab-2)(11ab+2)$.

(18) $(a+x+1)(a+x-1)$. (19) $(y+z+2x)(y+z-2x)$.

(20) $(a-b+c)(a-b-c)$. (21) $(x+y+a+b)(x+y-a-b)$.

(22) $(l+m-k-p)(l+m+k+p)$. (23) $(x+y+a-b)(x+y-a+b)$.

(24) $(x-y-a-b)(x-y+a+b)$. (25) $(x-y-a+b)(x-y+a-b)$.

(26) $(a-b+c)(a+b-c)$. (27) $(2z+x-y)(2z-x+y)$.

(28) $(8l-m+8)(8l+m-8)$. (29) $4xy$. (30) $(3x+5y)(x+y)$.

(31) $(3x-y)(x+y)$. (32) $20(x+y)(x-y)$. (33) $48ax$.

(34) $8x(3z-y)$. (35) 500000. (36) 2047.

(37) 1160. (38) 14076. (39) 2478600.

(40) 1004000. (41) $x(x+1)$. (42) $x(x-\frac{2}{3})$.

(43) $(a+\frac{3}{2})(a-\frac{1}{2})$. (44) $cd(cd+\frac{3}{2})$. (45) $(a+b+c)(a+b-c)$.

(46) $(a-c+b)(a-c-b)$. (47) $(x+y+z)(x-y-z)$.

(48) $(4a+b+2c)(4a-b-2c)$. (49) $(x+y-z)(x-y+z)$.

(50) $(x-y+2z)(x+y-2z)$. (51) $(3a-b+9x)(3a-b-9x)$.

(52) $(a-b-x+c)(a-b+x-c)$. (53) $(a-c+b)(a-c-b)$.

(54) $(3x-4z+t)(3x-4z-t)$. (55) $(x-y-b+c)(x-y+b-c)$.

(56) $(a^2-x^2-a+3)(a^2-x^2+a-3)$. (57) $(x^2+xy+y^2)(x^2-xy+y^2)$.

(58) $(a^2+3ab+9b^2)(a^2-3ab+9b^2)$. (59) $(x^2+4xy-11y^2)(x^2-4xy-11y^2)$.

Ex. 48.

(1) $(2x+1)(x-1)$. (2) $(2x-1)(x+1)$. (3) $(2x-1)(x+2)$.

(4) $(3y+2)(y-2)$. (5) $(5y+1)(y+5)$. (6) $(3-y)(1-2y)$.

(7) $(3+y)(1-2y)$. (8) $(3+2y)(1+2y)$. (9) $(6ab-5)(ab+1)$.

(10) $(2-x)(1+8x)$. (11) $(3-2x)(3+4x)$. (12) $(3a+1)(2a-3)$.
(13) $(4b+3)(b+5)$. (14) $(2+a)(1-2a)$. (15) $(2y-3)(3y+7)$.
(16) $(5c-1)(c-2)$. (17) $(6a-5)(5a-2)$. (18) $(3+2y)(5-6y)$.
(19) $(5-6y)(3-2y)$. (20) $(8-9x)(9-8x)$. (21) $(2a-5b)(a+4b)$.
(22) $(x-8)(3x-7)$. (23) $(9a-10c)(a-6)$. (24) $(8-3x)(9-24x)$.

Ex. 49.

(1) $(x+y)(x^2-xy+y^2)$. (2) $(x+2a)(x^2-2ax+4a^2)$.
(3) $(b-c)(b^2+bc+c^2)$. (4) $(x^3-y)(x^4+x^2y+y^2)$.
(5) $(3a+4b)(9a^2-12ab+16b^2)$. (6) $(2x-y)(4x^2+2xy+y^2)$.
(7) $(a-10b)(a^2+10ab+100b^2)$. (8) $(x-2)(x^2+2x+4)$.
(9) $(a+b-c)(a^2+2ab+b^2+ac+bc+c^2)$.
(10) $(x+y+z)(x^2+2xy+y^2-xz-yz+z^2)$.
(11) $(2zx-3)(4z^2x^2+6zx+9)$. (12) $(3-y)(9+3y+y^2)$.
(13) $(l^2+lm+m^2)(l^4-l^3m+3l^2m^2-lm^3+m^4)$.
(14) $(4-x)(16+4x+x^2)$. (15) $(2ay+3z)(4a^2y^2-6ayz+9z^2)$.
(16) $\left(\frac{1}{x}+\frac{1}{y}\right)\left(\frac{1}{x^2}-\frac{1}{xy}+\frac{1}{y^2}\right)$. (17) $\left(a+\frac{2}{b}\right)\left(a^2-\frac{2a}{b}+\frac{4}{b^2}\right)$.
(18) $\left(\frac{a^2b^2}{3}-\frac{3}{c}\right)\left(\frac{a^4b^4}{9}+\frac{a^2b^2}{c}+\frac{9}{c^2}\right)$. (19) $(6-xy)(36+6xy+x^2y^2)$.
(20) $\left(\frac{4}{a}-\frac{3}{b}\right)\left(\frac{16}{a^2}+\frac{12}{ab}+\frac{9}{b^2}\right)$.

Ex. 50.

(1) 6, 2. (2) 5, 4. (3) $\frac{1}{2}$, 3. (4) -2, 5. (5) 3, -3.
(6) $-4\frac{1}{2}$, 7. (7) 1, $-2\frac{1}{2}$, $\frac{1}{2}$. (8) 1, 1, -2. (9) 4, $3\frac{1}{2}$, $3\frac{1}{2}$.
(10) a, $\frac{-c}{b}$. (11) 0, -3, 2. (12) 0, 0, -1. (13) 3, 2.
(14) -3, -1. (15) 7, -5. (16) -6, 1. (17) $\frac{1}{2}$, -1.
(18) 8, 13. (19) $-\frac{1}{3}$, $\frac{2}{5}$. (20) $2a$, a. (21) 5, $-\frac{2}{3}$.
(22) 12, -12. (23) 0, 2. (24) 0, $-\frac{17}{2}$. (25) 0, 1, -1.
(26) $\frac{b}{2}$, $-\frac{b}{2}$. (27) 10, -2. (28) 0, 2, -2. (29) 0, -1, $\frac{1}{7}$.
(30) a, $\frac{b}{a}$. (31) $-\frac{p}{5}$, $5p$. (32) a^2, $-\frac{1}{a^2}$.

Ex. 51.

(1) $x^2-6x+5=0$. (2) $x^2-7x+12=0$ (3) $2x^2-5x+2=0$.
(4) $x^2-25=0$. (5) $x^2+4x-21=0$. (6) $4x^2+9x+2=0$.
(7) $x^3-4x^2+3x=0$. (8) $x^3-2x^2-x+2=0$. (9) $6x^2-x-2=0$.

(10) $x^2-a^2=0$. (11) $x^2-bx-2b^2=0$. (12) $x^2+8px+7p^2=0$.

(13) $x^2-2ax+a^2-b^2=0$. (14) $ax^2-a^2x-x+a=0$.

(15) $cx^2+2c^2x-2x-4c=0$. (16) $x^3-(a+b+c)x^2+(bc+ca+ab)x-abc=0$.

(17) $x^3+(a+b+c)x^2+(bc+ca+ab)x+abc=0$. (18) $x^3-3a^2x+2a^3=0$.

Ex. 52.

(1) $x-y$, $(x-y)^2(x+y)$. (2) $a+b$, $(a+b)(a-b)$.

(3) $a+b$, $(a+b)^2(a-b)$. (4) $x+3$, $x(x+1)(x+3)$.

(5) $x-1$, $(x-1)^2(x+1)$. (6) $a+4b$, $(a+4b)(a+3b)(a-b)(a-4b)$.

(7) None, $(x+a)^2(x-a)^2$. (8) $a-b$, $(a-b)^2(a^2+ab+b^2)$.

(9) $x+y$, $(x+y)^2(x-y)(x^2-xy+y^2)$. (10) xy, $xy(x+y)(x-y)(x^2+y^2)$.

Ex. 53.

(1) $\frac{x}{x+4}$. (2) 1. (3) $\frac{(x-y)^2}{x+y}$. (4) 1.

(5) $\frac{9x}{4}$. (6) $\frac{(x-1)(x+1)}{y}$. (7) $\frac{1}{x}$. (8) ab.

(9) $\frac{m-n}{(m+n)^2}$ (10) $\frac{(a+x)(x-y)}{2}$.

Ex. 54.

(1) ± 9. (2) ± 13. (3) ± 5. (4) ± 4.

(5) ± 3. (6) $\pm\sqrt{15}$. (7) $\pm\sqrt{7}$. (8) No real roots.

(9) 12, −4. (10) 4, −10. (11) 1, 0. (12) 3, −15.

(13) $7\frac{1}{2}$, $-2\frac{1}{2}$. (14) −3, −4. (15) −3, −4.

Ex. 55.

(1) 49, −9. (2) 6, −34. (3) 73, −1.

(4) 1, −33. (5) 30, 4. (6) 22, −4.

(7) 12·45..., −10·45.... (8) 18, 2. (9) 12, 19.

(10) −2, −24. (11) 3·37..., −2·37.... (12) No real roots.

(13) 14, 9. (14) 3, −10. (15) −·72, −2·78.

(16) 15, −12. (17) No real roots. (18) 7, −8.

(19) No real roots. (20) 1, $-\frac{5}{3}$. (21) 5, −4.

(22) 3, 4. (23) 7, −3. (24) −3, 1. (25) 4, 2.

(26) 3, $-\frac{1}{2}$. (27) 15, −12. (28) 11, 4. (29) 24·96, ·04.

(30) No real roots. (31) 21, 0. (32) $-\frac{1}{2}$, 4.

(33) $-\frac{1}{3}$, $-\frac{1}{2}$. (34) 1·15, −·48. (35) 1, $-\frac{3}{2}$.

(36) 3, $\frac{1}{3}$. (37) 4. (38) 9·33, ·67.

(39) (α) $\pm 1{\cdot}73$; (β) $\pm 1{\cdot}73$. (40) (α) 5, −1; (β) 6·46, −·46.

Ex. 56.

(1) $(x-4-\sqrt{7})(x-4+\sqrt{7})$. (2) $(x+2-\sqrt{5})(x+2+\sqrt{5})$.

(3) No real factors. (4) $(2x+7)(x-8)$.

(5) $(3x+2)(5x-7)$. (6) $(2x+1-\sqrt{3})(2x+1+\sqrt{3})$.

(7) $(3x+5-\sqrt{11})(3x+5+\sqrt{11})$. (8) No real factors.

(9) $\left(x+\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\left(x+\frac{1}{2}+\frac{\sqrt{5}}{2}\right)$. (10) $\left(x-\frac{1}{2}-\frac{\sqrt{5}}{2}\right)\left(x-\frac{1}{2}+\frac{\sqrt{5}}{2}\right)$.

(11) No real factors. (12) $(2x+y)(2x+7y)$.

(13) $\left(x-y\cdot\frac{1+\sqrt{13}}{2}\right)\left(x-y\cdot\frac{1-\sqrt{13}}{2}\right)$.

(14) $\left(5x+ay\cdot\frac{14+\sqrt{621}}{5}\right)\left(5x+ay\cdot\frac{14-\sqrt{621}}{5}\right)$. 15. No real factors.

(16) $(2x-11+\sqrt{2})(2x-11-\sqrt{2})$. (17) $(x+\sqrt{3})(x-\sqrt{3})$.

(18) $(x^2+2a^2)(x+\sqrt{2}\,.\,a)(x-\sqrt{2}\,.\,a)$. (19) No real factors.

(20) $(x+\sqrt[3]{10})(x^2-\sqrt[3]{10}\,.\,x+\sqrt[3]{100})$.

(21) $(x-\sqrt[3]{4}\,.\,a)(x+\sqrt[3]{4}\,.\,a+\sqrt[3]{16}\,.\,a^2)$.

(22) $(x-2\sqrt{b})(x+2\sqrt{b})(x^2+2\sqrt{b}\,.\,x+4b)(x^2-2\sqrt{b}\,.\,x+4b)$.

(23) $(3x^2+1)(\sqrt{3}\,.\,x+1)(\sqrt{3}\,.\,x-1)$.

(24) $(\sqrt{2x}+1)(\sqrt{2}\,.\,x-1)(x-\sqrt{3})(x+\sqrt{3})$.

Ex. 57. Corresponding values of x, y are bracketed together.

(1) $\{3, 6\}, \{-2, 1\}$. (2) $\{2\frac{2}{3}, 3\}, \{\frac{1}{3}, -4\}$.

(3) $\{-23, -10\}, \{4\frac{3}{5}, -\frac{4}{5}\}$. (4) $\{-4, -5\}, \{-\frac{2}{3}, 1\frac{2}{3}\}$.

(5) $\{5, 3\}, \{1, \frac{1}{3}\}$. (6) $\{-\frac{9}{10}, \frac{1}{2}\}, \{-\frac{3}{5}, 1\}$.

(7) $\{14, 9\}, \{-\frac{5}{2}, -2\}$. (8) $\{10{\cdot}71\ldots, -20{\cdot}13\ldots\}, \{2{\cdot}70\ldots, 3{\cdot}91\ldots\}$.

(9) $\{5, -1\}, \{\frac{31}{11}, \frac{25}{11}\}$. (10) $\{20, 7\}, \{-6, -1\}$.

(11) $\{11, -11\}, \{-3, 3\}$. (12) $\{3, 14\}, \{-\frac{331}{85}, \frac{1532}{85}\}$.

(13) $\{4, 1\}, \{3, -1\}$. (14) No real roots.

(15) $\{4{\cdot}32\ldots, 1{\cdot}64\ldots\}, \{2{\cdot}67\ldots, -1{\cdot}66\ldots\}$. (16) $\{56, 22\}, \{26\frac{2}{3}, 0\}$.

(17) $\{9, 2\}, \{2, 9\}$. (18) $\{13, 1\}, \{-1, -13\}$.

(19) $\{8, 7\}, \{-7, -8\}$. (20) $\{7, -2\}, \{-2, 7\}$.

(21) $\{14{\cdot}15\ldots, {\cdot}85\ldots\}, \{{\cdot}85\ldots, 14{\cdot}15\ldots\}$. (22) No real roots.

(23) $\{4, 6, 1\}, \{-4, -6, -1\}$. (24) $\{-1, 3, 0\}, \{\frac{1}{3}, 1\frac{2}{3}, 2\frac{2}{3}\}$.

(25) $\{1, \frac{3}{2}\}, \{1, \frac{3}{2}\}, \{-1, \frac{1}{2}\}, \{-2, 3\}$.

(26) $\{3, 0\}, \{9, 2\}, \{9, -3\}, \{3, -1\}$.

Ex. 58.

(1) 48. (2) 12 or -8. (3) 37, 38. (4) 60, 62.
(5) 39, 41. (6) 8, 10. (7) 12, 15. (8) ± 22.
(9) ± 65. (10) 13 ft., 7 ft. (11) 21 ft., 16 ft. (12) 42 ft.

Ex. 59.

(1) ·62 ft. (2) ·73 ft. (3) ·69 ft. (4) ·59 ft.
(5) $\frac{1}{3}$ ft. (6) $\frac{5}{8}$ ft. (7) 52. (8) 36.
(9) 30, 40. (10) 24, 25. (11) 39·64, 5·36.

Ex. 60.

(10) 2·62, ·38; 3·56, $-$·56; acc. as $c<$, $=$, or $>\frac{9}{4}$.
(11) $-1·27$, $-4·73$. (13) ·62, $-1·62$. (14) 1, 1.
(15) (i) ·62, $-1·62$; (ii) 1·37, $-$·37; (iii) 1·82, $-$·82.

Ex. 61.

(2) $-a^4-b^4-c^4+2a^2b^2+2b^2c^2+2c^2a^2$. (3) $x^6+x^4y^2-x^2y^4-y^6$.

(4) $a^2+\frac{b^2}{4}+\frac{c^2}{9}+\frac{d^2}{16}-ab+\frac{2ac}{3}-\frac{ad}{2}-\frac{bc}{3}+\frac{bd}{4}-\frac{cd}{6}$. (5) 0.

(6) (i) $(x-1)(x+1)(x^2+1)(x^4+1)$; (ii) $3b(x-y)(x+y)(x^2+y^2)$;
(iii) $(x-z)(x+z)(x^2+xz+z^2)(x^2-xz+z^2)$.

(7) (i) $3a(a-2c^2)(a+2c^2)(a^2+4c^4)$; (ii) $x(x+1)^2(x-1)$;
(iii) $(a^p-b^q)(a^p+b^q)(a^{2p}+b^{2q})$.

(8) (i) $(a+b+c-d)(a+b-c+d)(-a+b+c+d)(a-b+c+d)$;
(ii) $(3x-y)(3x+y)(9x^2+y^2)$.

(9) (i) $(2x-y)(2x+y)(4x^2+2xy+y^2)(4x^2-2xy+y^2)$;
(ii) $(x-y)(x+y)(x-3y)(x+3y)$; (iii) No real factors.

(10) (i) $(a+b+c)(a+b-c)(-a+b+c)(a-b+c)$;
(ii) $(2x+3y)(3x-4y)$.

(11) (i) 2; (ii) $\frac{7}{11}$; (iii) -1, 2; or $\frac{8}{7}$, $-\frac{17}{14}$.

(12) (i) 1 or $4\frac{1}{3}$; (ii) 3; (iii) 2, 1; or $-\frac{22}{7}$, $\frac{25}{7}$.

(13) (i) -47; (ii) 145; (iii) 4; (iv) 0.

(14) (i) $x^2-10x+21=0$; (ii) $x^2-5x-24=0$; (iii) $2x^2-11x+5=0$;
(iv) $x^2+10x+24=0$; (v) $x^2-(a+b)x+ab=0$;
(vi) $x^2-2ax+a^2-b^2$; (vii) $x^2-a^2-b^2+2ab=0$.

(15) 2, 2; or $\frac{2}{5}$, $2\frac{4}{5}$. (17) 7·85 in., 1·15 in.

(18) 30 half crowns, 10 shillings. (19) 40 mi./hr. 20. 30s.

Ex. 63.

(1) $30\frac{2}{5}$. (2) 4000. (3) £650. (4) $4\frac{1}{2}$ years. (5) £150.
(6) £260. (7) 4%. (8) 2 ft. (9) 6 ft.

Ex. 64.

(1) ·7 in. (2) 2·83 in. (3) 1·41 in. (4) 8 in.
(5) ·5 in. (6) 2·4 in., ·4 in. (7) 1·73 in.
(8) (i) (α) $1\frac{1}{3}$ ft., (β) 4 ft.; (ii) (α) 1·17 ft., (β) 6·83 ft.
(9) (i) $AP=7\cdot2$ in.; (ii) $AQ=4$ in. (10) 3, 4, 5;
(11) (i) 625 sq. ft.; (ii) 1717 sq. ft. (12) 104 ft., 94 ft.
(13) 40 yds., 121 yds. (14) 15 yds., 8 yds. (15) 4·717 in.
(16) (i) 4·33 in.; (ii) 10·8 sq. in.
(17) (i) (α) 3·873 in., (β) 4·472 in.; (ii) (α) 3·708 in., (β) 4·472 in.
(18) 3·317 in. (19) 12 sq. in. (20) $8\frac{1}{4}$. in.
(21) (i) $BN=72$; (ii) $AN=30$; (iii) area $=1680$.
(22) (i) $BN=\dfrac{a^2+c^2-b^2}{2a}$; (ii) $AN=\dfrac{\sqrt{P}}{2a}$ where
$P=2b^2c^2+2c^2a^2+2a^2b^2-a^4-b^4-c^4$; (iii) area $=\frac{1}{4}\sqrt{P}$.

Ex. 65.

(1) 320 miles. (2) $\frac{5}{9}$. (3) $\frac{14}{29}$. (4) $\frac{9}{14}$.
(5) 15 pence a doz. (6) 2s. 9d. a doz. (7) $\frac{1}{4}$, $\frac{3}{8}$. (8) $\frac{13}{18}$.
(9) 2264. (10) 165. (11) 60 half-crowns, 22 shillings.
(12) 660 yards. (13) 660 yards. (14) 3 yds., 4 yds.
(15) 32. (16) A by 5 votes. (17) 15 years. (18) 6 miles.
(19) 12 mi./hr. (20) $\frac{1}{5}$. (21) 1 mile 88 yds., $3\frac{1}{2}$ mi./hr.
(22) $6\frac{3}{4}$. (23) $9\frac{7}{27}$. (24) 15 mins., 20 mins. (25) 12355, 4942.
(26) 4311, 2874. (27) 11 miles. (28) 5 miles. (29) $8\frac{1}{6}$ miles.
(30) 20, 13. (31) Sheep £4., cow £12. (32) 3 per cent., 7 per cent.
(33) £500, £200. (34) 4, $3\frac{1}{2}$. (35) 189 sq. ft., 3 ft. wide.
(36) $9\frac{3}{4}$ ft. (37) 6 days, 5 days. (38) 8 days, 6 days.
(39) $15\frac{1}{2}$ mi./hr., $4\frac{1}{2}$ mi./hr. (40) 2 mi./hr.
(41) 10 mi./hr., 5 mi./hr. (42) $5\frac{1}{2}$ ft., 4 ft. (43) 4 yds., $3\frac{1}{2}$ yds.
(44) 50, 60. (45) $\frac{3}{11}$, $\frac{5}{22}$ pints. (46) 10 gals., 9 gals.
(47) 1, 2, 3, 4 or 7, 8, 9, 10. (48) 17 men, 21 men. (49) 144.
(50) $3\frac{1}{2}$d. (51) 16d. (52) 15. (53) 84. (54) 12s. 6d.
(55) 21s. (56) 10 mi./hr. (57) $2\frac{1}{4}$ hrs. (58) $6\frac{3}{4}$ sq. ft.

(59) 8748 sq. yds. **(60)** $33\frac{3}{4}$ sq. ft. **(61)** 9 in. **(62)** 9 in. sq.
(63) 18 ft. by 24 ft. **(64)** 16 in. by 4 in. **(65)** 16 in. by 4 in.; 8 in. sq.
(66) 144 yds. **(67)** 80 ft., 36 ft. **(68)** $1\frac{1}{2}$ in. **(69)** 37 ft.
(70) 10 mins., 8 mins. **(71)** Side 6 ft. **(72)** 12 ft. × 6 ft. × 3 ft.
(73) $10\frac{1}{2}$ ft., 12 ft. **(74)** 10 mi./hr. **(75)** 2 hrs. or 6 hrs.
(76) 6 miles. **(77)** 12 mi./hr., 10 mi./hr.
(78) 20 mi./hr., 15 mi./hr. **(79)** 45 mi./hr., 30 mi./hr.
(80) 40, 60. **(81)** 81. **(82)** 63. **(83)** $\frac{6}{7}$. **(84)** 89.
(85) 45. **(86)** 17, 15. **(87)** 16, 7. **(88)** 5929, 77.
(89) $4\frac{1}{2}$ mi./hr., 4 mi./hr. **(90)** 9 in., 12 in. **(91)** 36.
(92) 11432 sq. yds. **(93)** 1, 2, 3, 4 or 5, 6, 7, 8. **(94)** 2 hrs. $21\frac{2}{3}$ mins.
(95) 30 ft. **(96)** 576. **(97)** 5 mi./hr. **(98)** 5, $4\frac{8}{9}$.
(99) £6000, £4000. **(100)** 180,000. **(101)** 10 gals.
(102) 1 tea sp. fl. **(103)** 1s. 6d., 1s. 9d. **(104)** 30s.
(105) $1\frac{2}{3}$ oz., $9\frac{1}{3}$ oz. **(106)** $1\frac{1}{2}$. **(107)** 4860; *B*, *C*, *A*, *D*, *E*.
(108) 13s., 11s. **(109)** 9, 8. **(110)** 25s, 12s.

Ex. 66.

(1) (i) $2\sqrt{2}$; (ii) $9\sqrt{2}$; (iii) $3\sqrt{3}$; (iv) $2\sqrt[3]{4}$; (v) $3\sqrt[3]{3}$;
(vi) $8\sqrt{5}$; (vii) $7\sqrt[3]{2}$; (viii) $2\sqrt[5]{2}$; (ix) $30\sqrt{2}$; (x) $20\sqrt{3}$.

(2) (i) 2·828; (ii) 12·73; (iii) 5·196.

(3) (i) 3·240; (ii) 14·77; (iii) −1·73; (iv) 12·57.

(4) (i) $\sqrt{6}$, 2·449; (ii) $\sqrt{35}$, 5·916; (iii) 16; (iv) $7\sqrt{3}$, 12·12; (v) 3;
(vi) 7; (vii) $\sqrt{2}$, 1·414; (viii) $2\sqrt{3}$, 3·464; (ix) $11\sqrt{2}$, 15·56;
(x) $7\sqrt{6}$, 17·15; (xi) $\frac{9}{5}\sqrt{5}$, 4·02; (xii) $\frac{\sqrt{2}}{20}$, ·0707; (xiii) $\frac{\sqrt{6}}{2}$, 1·224;
(xiv) $\frac{\sqrt{2}}{10}$, ·1414; (xv) 8.

(5) (i) a^2; (ii) a^3x^3; (iii) $\frac{a}{b}$; (iv) $a^2b^2c^2$; (v) $abx^3\sqrt{ab}$; (vi) $\frac{a\sqrt{ax}}{x^2}$;
(vii) $abc\sqrt{abc}$; (viii) xyz; (ix) ab; (x) $xy\sqrt[3]{x}$.

Ex. 67.

(1) (i) $7-\sqrt{35}$, 1·084; (ii) $2+\sqrt{2}$, 3·414; (iii) $6+\sqrt{2}$, 7·414; (iv) 2;
(v) 1; (vi) 11; (vii) $5+2\sqrt{6}$, 9·899; (viii) 68; (ix) $9-4\sqrt{2}$, 3·343.

(2) (i) $ax+x^2$; (ii) $x+\sqrt{xy}$; (iii) $x-a$; (iv) ab^2-a^2b; (v) $x-y$;
(vi) $x+y+2\sqrt{xy}$; (vii) $2x+2\sqrt{x^2-y^2}$; (viii) $2a^2-2\sqrt{a^4-1}$;
(ix) $(a^2-b^2)x-(a^2+b^2)y$; (x) $2aby+(a^2-b^2)\sqrt{x^2-y^2}$.

(3) (i) $\frac{\sqrt{5}+\sqrt{3}}{2}$, 1·984; (ii) $\frac{\sqrt{3}-1}{2}$, ·366; (iii) $\sqrt{7}+\sqrt{5}$, 4·882;
(iv) $\sqrt{3}-1$, ·732; (v) $\frac{5}{4}(\sqrt{5}-1)$, 1·545; (vi) $6+\sqrt{35}$, 11·916;
(vii) $-\frac{11+2\sqrt{10}}{9}$, −1·925; (viii) $\frac{22+5\sqrt{2}}{31}$, ·938.

(4) (i) $\frac{a-\sqrt{x}}{a^2-x}$; (ii) $\frac{a-\sqrt{ab}}{a-b}$; (iii) $\frac{x+y-2\sqrt{xy}}{x-y}$;
(iv) $\frac{ax+b^2\sqrt{ax}+bx\sqrt{x}+ab\sqrt{a}}{x^2-ab^2}$; (v) $\frac{\sqrt{a+b}+\sqrt{a-b}}{2b}$;
(vi) $\frac{1-\sqrt{1-a^4}}{a^2}$; (vii) $\frac{x^2\sqrt{x^2+y^2}-x^2\sqrt{x^2-y^2}}{y^2}$;
(viii) $\frac{a^3-ac-bc-(a^2-b+c)\sqrt{b}-(a^2+b-c)\sqrt{c}+2a\sqrt{bc}}{a^4-2a^2b-2a^2c+b^2+c^2-2bc}$;

(5) (i) $\sqrt{7}+\sqrt{5}$; (ii) $\sqrt{3}-\sqrt{2}$; (iii) $\sqrt{6}+\sqrt{3}$, 4·181; (iv) $\sqrt{6}+\sqrt{3}$;
(v) $3+\sqrt{2}$, 4·414; (vi) $\sqrt{7}-1$; (vii) $\sqrt{7}+\sqrt{2}$; (viii) $\sqrt{13}-\sqrt{10}$.

Ex. 70.

(1) (i) 2·828; (ii) 54; (iii) 3·873; (iv) ·177; (v) 66; (vi) 109·56.

(2) and (4) $10^{\frac{1}{6}}=1·154$, $10^{\frac{1}{3}}=1·334$, $10^{\frac{1}{4}}=1·778$, $10^{\frac{3}{8}}=2·371$, $10^{\frac{1}{2}}=3·162$, $10^{\frac{5}{8}}=4·217$, $10^{\frac{3}{4}}=5·623$, $10^{\frac{7}{8}}=7·499$.

(5) $10^{\cdot2}=1·585$, $10^{\cdot4}=2·512$, $10^{\cdot6}=3·981$, $10^{\cdot8}=6·310$, $10^{\cdot2}=1·698$.

(6) $2=10^{\cdot301}$, $3=10^{\cdot477}$, $5=10^{\cdot699}$, $7=10^{\cdot845}$, $4·26=10^{\cdot629}$, $1·74=10^{\cdot240}$, $5·83=10^{\cdot766}$.

(7) $20=10^{1·301}$, $3000=10^{3·477}$, $·5=10^{-1+·699}=10^{-·301}$, $07=10^{-2+·845}=10^{-1·155}$, $4260=10^{3·629}$, $174=10^{2·240}$, $·00583=10^{-3+·766}=10^{-2·234}$.

(9) $10^{\frac{1}{2}}=3·162$, $10^{2·427}=416·9$, $10^{\frac{7}{5}}=5·995$, $10^{-1·213}=·06124$, $10^{1·08}=12·02$.

Ex. 73.

(1) (*a*) 17450, (*b*) 2·030, (*c*) 65060, (*d*) 448·5, (*e*) 2,044,000,000, (*f*) 71·86, (*g*) 2925.

(2) (*a*) 59,910,000, (*b*) 6969, (*c*) 223,400,000, (*d*) 1,540,000, (*e*) $7·017\times10^{12}$, (*f*) 246,700, (*g*) 10,040,000.

(3) (*a*) ·04575, (*b*) ·000,005,322, (*c*) ·1706, (*d*) ·001,176, (*e*) 5358, (*f*) ·000,188,4, (*g*) ·007,670.

(4) (*a*) 537·9, (*b*) ·06258, (*c*) 2005, (*d*) 13·82, (*e*) 62,990,000, (*f*) 2·215, (*g*) 90·18.

(5) (*a*) 2014, (*b*) ·2348, (*c*) 7525, (*d*) 51·88, (*e*) 236,300,000, (*f*) 8·312, (*g*) 338·4.

(6) (*a*) 31·99, (*b*) ·003,720, (*c*) 119·2, (*d*) ·8221, (*e*) 3,745,000, (*f*) ·1317, (*g*) 5·362.

(7) (*a*) 1·170, (*b*) ·000,136,1, (*c*) 4·363, (*d*) ·030,08, (*e*) 137,100, (*f*) ·004,819, (*g*) ·1962.

(8) $1{\cdot}845 \times 10^{19}$. (9) £2936, £3747, £4783, £6105, £7793.

(10) £1340. (11) (i) (*a*) 24, (*β*) 38 years; (ii) (*a*) 15 (*β*) 23 years.

Ex. 74.

(1) $x^4y+x^3y^2+x^2y^3+xy^4$, $2x+2y$. (2) 96, 45, 0, −27.

(3) $\Sigma x^2 = x^2+y^2+z^2+w^2$, $\Sigma yz = xy+xz+xw+yz+yw+zw$, $\Sigma xyz = yzw+zwx+wxy+xyz$.

(4) $a^2+b^2+c^2-bc-ca-ab$, $a^3+b^3+c^3-b^2c-c^2a-a^2b$, $a^3b^2c+b^3c^2a+c^3a^2b$.

(6) $b=c$. (7) $\Sigma x^2(b^2-c^2)$. (9) $\Sigma x(b^2-c^2)$.

(10) $-\Sigma x(y^2+z^2)-4xyz$.

Ex. 76.

(1) $A=1$, $B=3$, $C=1$. (2) $a=11$, $b=-6$, $c=-3$, $d=2$.

(3) $A=-1$, $B=2$. (4) $A=1$, $B=0$, $C=-3$.

(5) $A=0$, $B=0$, $C=6$, $D=-6$, $E=1$. (6) $l=-2$, $m=3$, $n=-2$.

(7) $a=2$, $b=-1$, $c=1$, $d=0$. (8) $A=1$, $B=3$.

(9) $A=1$, $B=3$, $C=6$. (10) $\Sigma a^3+3\Sigma a^2b+6\Sigma abc$.

Ex. 77.

(1) −2, 2, 18. (2) 3, −1, 195.

(3) $4a+2b+c$, ay^2+by+c, $ax^2+(2a+b)x+a+b+c$.

(4) n^2+3n+3, n^2-n+1. (5) $2a^3$, $3a^3$, $6a^3$, $11a^3$. (8) $6an$.

Ex. 78.

(1) 1, 7, 3, 3, 7. (2) 0, 62, 17, 2, −3. (3) 0, 80, 15, 0, 15.

(4) −12, 0, −12, 0, 0. (5) −6, 20, 0, 0, 0. (6) $4a^2$, 0, $16a^2$.

(7) $-24a^3$, $-8a^3$, 0. (8) 0, $-2a^9$, $19683a^9$. (9) $27y^3$, 0, $-y^3$.

(10) $63y^6$, 0, $63y^6$. (11) 0, $-33y^5$, $-64y^5$.

(15) −1, $-(a+b+c)$, $-(a^2+b^2+c^2+bc+ca+ab)$.

(16) $2a+2b$, $a+b=0$. (18) No. (19) $p+q=0$.

Ex. 79.

(1) (i) $a+b+2\sqrt{ab}$; (ii) $a+b-2\sqrt{ab}$.

(2) (i) $\dfrac{a^4}{(a-b)(a-c)}+\dfrac{b^4}{(b-c)(b-a)}+\dfrac{c^4}{(c-a)(c-b)}$;

(ii) $\dfrac{a^4+b^2c^2}{(a-b)(a-c)}+\dfrac{b^4+c^2a^2}{(b-c)(b-a)}+\dfrac{c^4+a^2b^2}{(c-a)(c-b)}$.

(3) Of the first only. (4) $(\Sigma a)^2 = \Sigma a^2 + 2\Sigma ab$.

(5) $s - a = \frac{b+c-a}{2}$. (6) $x^{2p+mq-nr}$. (7) $a = 4$.

(8) (i) $\sqrt{7} - \sqrt{3}$; (ii) $\sqrt{9} - \sqrt{2}$. (9) 1, 64, 4, -4, $\frac{1}{8}$, $\frac{1}{16}$.

(10) $c = 12$. (11) 90. (12) $19 = 10^{1\cdot2788}$, $426\cdot6 = 10^{2\cdot63}$.

(14) $1 + \sqrt{3} \,.\, x + 2x^2 + \sqrt{3} \,.\, x^3 + x^4$. (15) $3\sqrt{3} - 2\sqrt{2}$.

(16) $a = 4$, $b = -4$, $c = -\frac{1}{4}$. (18) (i) $\frac{2}{a}$; (ii) $\frac{2 + \sqrt{1-a} - \sqrt{1+a}}{a}$.

(19) (i) 1·585; (ii) 25·12; (iii) 398·1.

(20) $n = (n+1)^3 - 3(n+1)^2 + 3(n+1) - 1$.

(21) $ax^4 + (a^2 + 2a + b + 1)x^3 + (2ab + a + 2b + c + 2)x^2$
$+ (ac + b^2 + b + 2c)x + bc$, $x^2 + (a+2)x + b$.

(22) $x^3 - x + \frac{1}{x} - \frac{1}{x^3}$. (23) $(x+1)(x-1)^2(x^2 - x + 1)$.

(24) $\frac{(a+2)(a^2+x^2)}{(a-x)(a^2 - ax + x^2)}$. (26) $\frac{(a+b-c)(b+c-a)(c+a-b)}{(a+b+c)^3}$.

(27) (i) $-\frac{3}{2}$ or $\frac{5}{2}$; (ii) 0 or 4. (28) $x - 1$.

(29) A 148, B 296. (30) A £3000, B £4500.

(31) (i) $1 + 2x + 2x^2 + 2x^3 + 2x^4$;
(ii) $1 + (a+b)x + a(a+b)x^2 + a^2(a+b)x^3 + a^3(a+b)x^4$.

(32) $a + a^{\frac{2}{3}}b^{\frac{1}{3}} - 2b$. (33) $4t^4 + 7t^3 - 23t^2 + 22t - 15$ where $t = x + 1$.

(35) £800, £200. (36) $-(b-c)(c-a)(a-b)$. (37) $-8m^3$.

(39) $b = 0$ or 9; x or $x + 3$. (40) 72.

Ex. 80.

(1) (i) $y = ax$; (ii) $y = \frac{a}{x}$; (iii) $y = ax^2$; (iv) $y = \frac{ax}{z^2}$; (v) $z = axy^2$;
(vi) $y = a + bx$; (vii) $y^2 = ax^3$.

(3) 12. (4) 27. (5) $a = 14$, $b = -1$; -6. (6) 16.

(7) $6\frac{1}{4}$. (8) $12\frac{1}{4}$. (9) $a = 12$, $b = 2$; 9. (10) (i) no; (ii) yes.

(11) 132·7 sq. in. (12) 7·8 in. (13) $\frac{1}{3}$ in.

(14) 130·9 cub. ft. (15) 3 ft. 6·3 in. (16) 10stone 12·3lbs.

(17) £2, £18. (18) £1150. (19) The latter by £164.

Ex. 82.

(1) $x + 7$. (2) $x - 3$. (3) $x + 7$. (4) $y + 1$. (5) $a^2 + 3a + 2$.

(6) $3x^2 - 4xy - y^2$. (7) $27x^6 - 9x^5 - 21x^4 + 20x^3 - 7x^2 - 2x + 1$.

(8) $x^5 - 13x^4 + 67x^3 - 161x^2 + 172x - 66$.

(9) $6x^5-5x^4+17x^3+15x^2+x+14$. (10) $x^5-8x^6+3x^5+x^3-8x+3$.

(11) $a^4-3a^3+3a^2+12a-28$. (12) $x-3$. (13) $x-1$. (14) None.

(15) $\frac{x^2-3x+2}{x^2-x+1}$. (16) $\frac{6x^2-11x+4}{2x^2-9x+9}$.

(17) $\frac{x^2-3x+5}{2x^2-3x+4}$. (18) $\frac{5y^3-2y+7}{7y^2+2y-5}$.

(19) x^3-2x^2+x. (20) x^2-4x+1. (21) $x-1$.

Ex. 83.

(1) $\frac{2(x+4)}{(x+5)(x+3)}$. (2) $\frac{2}{(x+7)(x+9)}$. (3) $\frac{5-x}{(3x+1)(x+3)}$.

(4) $\frac{5x^2+12x+1}{2(2x+1)(x+3)}$. (5) $\frac{a^2-2ab-b^2}{a^2-b^2}$. (6) $\frac{3x-2}{(x-2)^2}$.

(7) $\frac{1}{2(a-b)}$. (8) $\frac{2x+3}{(x+1)(x+2)(x-2)}$. (9) $\frac{x+5}{(x-3)(x-4)(x+1)}$.

(10) $\frac{x^2+4x+1}{(x-1)^2(x+1)}$. (11) $\frac{5x-2y}{2(x^2-y^2)}$. (12) $\frac{13a^2-11ab+2b^2}{3(a^2-b^2)}$.

(13) $\frac{-2y}{(x+y)(x-3y)}$. (14) $\frac{2}{(x+1)(x+2)(x+3)}$.

(15) $\frac{11x^2-26x+14}{(5x-1)(4x-3)(3x-2)}$. (16) $\frac{a(2x^2+5x-7)}{(2x+1)(x+1)(x-4)}$.

(17) $\frac{2}{(2-a)(3-a)(4-a)}$. (18) $\frac{8x}{3a+2x}$. (19) $\frac{b}{a-b}$.

(20) $\frac{-6ax(x^2-3a^2)}{x^4-81a^4}$. (21) $\frac{7x+6y}{3x(x-y)}$. (22) $\frac{10(a+b)b^2}{(a-2b)(a+3b)(a+4b)}$.

(23) 2. (24) $\frac{20}{x^2-1}$. (25) $\frac{2}{b-3a}$.

(26) $\frac{2ab(a^2+b^2)}{a^6-b^6}$. (27) $\frac{4(a^4x^3-b^4y^3)}{a^4x^4-b^4y^4}$. (28) $-\frac{2}{x-3}$.

(29) $\frac{x^2-x+1}{x^2-5x+6}$. (30) $\frac{4x^2+3x-11}{(x^2-4)(x^2-1)}$. (31) $\frac{2(x+1)}{(x-3)(x-1)}$.

(32) $\frac{ab(x^2-1)}{(a^2-b^2)x}$. (33) 1. (34) $\frac{b+c+d-a}{2}$.

(35) $\frac{y(x^3+y^3)}{x(x^3-y^3)}$. (36) 1. (37) $\frac{-1}{a+b}$.

(38) $\frac{a^2+2ab-b^2}{a^2-2ab-b^2}$. (39) $\frac{1}{x^2+y^2}$. (40) m.

(41) 0. (42) d. (43) -3.

(44) 0. (45) 1. (46) 0.

(47) $\dfrac{\Sigma yz - \Sigma x^2}{(y-z)(z-x)(x-y)}$. (48) $\dfrac{-\Sigma x^3 + \Sigma y^2 z - 3xyz}{(y-z)(z-x)(x-y)}$.

(49) 1. (50) $\dfrac{2\Sigma bc + 2x\Sigma a}{(b+c)(c+a)(a+b)}$. (51) 1.

(52) Σa. (53) $\dfrac{3x - \Sigma a}{(x-a)(x-b)(x-c)}$. (54) $2\Sigma x$.

Ex. 84.

(1) $\dfrac{t-q}{p-r}$. (2) $a-b$. (3) $\dfrac{ab}{a+b}$. (4) $\dfrac{2ab}{a+b}$.

(5) $\pm a$. (6) a, b. (7) a, b. (8) $\dfrac{r}{p-q}$, $\dfrac{-r}{p-q}$.

(9) 0, 1. (10) $2b$, $2b-a$; b, $b-a$.

Ex. 85.

(1) $\frac{1}{7}$. (2) 0, 5. (3) 4, $1\frac{9}{43}$. (4) 2.

(5) $-1 \pm \sqrt{-2}$, 0. (6) 0, 6·82, 4·18. (7) 11, $-\frac{2}{5}$. (8) 3, $\frac{1}{2}$.

(9) ± 3, ± 1; $\mp 3\sqrt{7}$, $\pm\sqrt{7}$. (10) 0, $\dfrac{a}{2}(1 \pm \sqrt{-3})$.

(11) $a^2 + ab + b^2$. (12) $\dfrac{cd(a+b) - ab(c+d)}{ab - cd}$.

(13) $a+b$, $a-b$. (14) $-\dfrac{2ab}{a+b}$. (15) 1.

(16) 0. (17) $\pm\sqrt{-ab}$. (18) $-\dfrac{a^2}{b}$, $\dfrac{-b^2}{a}$.

(19) $x = \dfrac{(a+b-c)(a-b+c)}{2bc}$, etc. (20) c, 0, a.

Ex. 86.

(1) $x+3$. (2) $x-2y$. (3) $4a+3b$.

(4) $5x-y$. (5) $1-7ab$. (6) x^2+8y^2.

(7) x^2-3x+4. (8) $3a^2-2ab+5b^2$.

(9) $2x^3-3x^2-4x+5$. (10) x^3-2x^2-2x-1.

(11) $7y^3-8y^2+2y-1$. (12) $5x^3-3x^2y+y^2$.

(13) $a^3-2a^2b+ab^2-3b^3$. (14) $3x^3-x+4$.

(15) $2x^3-3x^2y+5xy^2-4y^3$. (16) $2a^2-3a^{-1}+a^{-2}$.

(17) $\dfrac{1}{2}-2x+3x^2-\dfrac{1}{3}x^3$. (18) $a^{-\frac{2}{3}}+2b^{-\frac{1}{3}}c^{\frac{2}{3}}+3a^{\frac{1}{3}}c^{-1}$.

(19) 1, 2. (21) $a=53$, $b=-28$, $c=-7$.

(22) $a=2$, $b=-1$; four coefficients: three conditions.

(23) $2x^2+5-\dfrac{7}{x^2}$. (24) x^2+3x-y^2, $a=6$.

Ex. 88.

(1) 185,1640. (2) 75, 1440. (3) $-34, -18$. (4) $n, \frac{n(n+1)}{2}$.

(5) $2n-1, n^2$. (6) $(3n-2)x, \frac{n(3n-1)x}{2}$.

(7) $\frac{13n-4}{4}, \frac{n(13n+5)}{8}$. (8) $\frac{3a}{14}(n+1), \frac{3an(n+3)}{28}$.

(9) $(n-1)b-(n-2)a$. (10) $n+1$ terms; $\frac{n+1}{2}(2a+nx)$. (11) $(2n+1)a$.

(12) $2na$. (13) 1431, the alternate terms form an A.P. (14) 1591.

(15) (i) a; (ii) $\frac{a+b}{2ab}$; (iii) $\frac{x+y}{2}$. (16) $\frac{1}{3}, \frac{1}{6}, 0, -\frac{1}{6}, -\frac{1}{3}$.

(17) 18·4, 17·6, 16·8,... 6·4. (18) $a+\frac{s(b-a)}{n+1}$ where $s=1, 2, 3, \ldots n$.

(19) 18, 16, 14, ... (20) 0, 4, 8, ... (21) $-120, -115, -110, \ldots$.

(22) $3\frac{8}{9}, 4\frac{6}{9}, 5\frac{4}{9}, \ldots$. (23) 11. (24) 14. (25) 23.

(26) 20 or 21. (27) 9 or 20. (28) $-12, -9, -6, \ldots 6$.

Ex. 90.

(1) $\frac{3^{n-3}}{2^{n-4}}$. (2) $(-1)^{n-1}.\frac{2}{3^{n-1}}$. (3) $\frac{x^{n-6}}{a^{n-6}}$. (4) $\frac{x^{\frac{n-1}{2}}}{a^{n-2}}$.

(5) $9\frac{507}{512}$. (6) $18\frac{83}{128}$. (7) $364\frac{40}{81}$. (8) $182\frac{20}{81}$.

(9) $\frac{1-x^{n+1}}{1-x}$. (10) $\frac{1-(2x)^{20}}{1+2x}$. (11) $\frac{1-x^{20}}{(1-x)^2}-\frac{19x^{20}}{1-x}$.

(12) $\frac{2-147x^{30}}{1-x}+\frac{5x(1-x^{29})}{(1-x)^2}$. (13) $\frac{a-nax^n}{1-x}+\frac{ax(1-x^{n-1})}{(1-x)^2}$.

(14) $\frac{a(1+73x^{24})}{1+x}-\frac{3ax(1-x^{24})}{(1+x)^2}$. (15) $\frac{2}{9}$. (16) $\frac{71}{99}$. (17) $\frac{37}{60}$.

(18) $2\frac{5}{99}$. (19) $1\frac{59985}{249975}$. (20) $8{\cdot}15\times10^{17}$. (21) $2-1{\cdot}58\times10^{-30}$.

(22) $4{\cdot}17\times10^{8}$. (23) $\frac{10}{17}-1{\cdot}90\times10^{-16}$. (24) About £17480.

Ex. 92.

(4) $\frac{1}{4n+19}$. (5) $\frac{1}{n(x+1)-1}$. (6) $\frac{4}{9}, \frac{2}{5}, \frac{4}{11}$.

(7) $2\frac{2}{11}, 2\frac{2}{5}, 2\frac{2}{3}$. (8) (i) $2y-x$; (ii) $\frac{y^2}{x}$; (iii) $\frac{xy}{2x-y}$.

(9) $A=\frac{a+b}{2}$, $G=\sqrt{ab}$, $H=\frac{2ab}{a+b}$. (10) $\frac{b^2}{2ab-a^2}$. (11) 10, 12, 14.

(12) 16, 20, 24, 28. (13) 12, 3, $\frac{3}{4}$. (14) 2680 yds. (15) 5 hours.

Ex. 93.

(3) 56. (5) 9900, 5040, 6840. (6) 120, 40320. (8) $r=n$.

(9) 176. (10) $n(n-1)(n-2)$, $\overline{n-4}!$

Ex. 94.

(3) 190, 120, 55, 364. (4) 364, 20. (5) 18. (6) 16 or 5.

(7) 4620. (8) 1440, 14400.

Ex. 95.

(2) 840, 86,486,400, 34650. (3) 125,000, one in a million.

(6) 126. (8) 720. (9) 60, 120, 20, 325. (10) 6, 6.

(11) 2, 4, 8, 16. (12) 2880, 576. (13) 9. (14) 19 !, 18 . 19 !

Ex. 96.

(1) $a^5+5a^4x+10a^3x^2+10a^2x^3+5ax^4+x^5$.

(2) $a^7+7a^6b+21a^5b^2+35a^4b^3+35a^3b^4+21a^2b^5+7ab^6+b^7$.

(3) $a^6-6a^5b+15a^4b^2-20a^3b^3+15a^2b^4-6ab^5+b^6$.

(4) $16x^4-32x^3y+24x^2y^2-8xy^3+y^4$.

(5) $2x^6+120x^4+480x^4+128$. (6) $24x^3+\frac{216}{x^3}$. (7) 66.

(8) $-672x^6$. (9) $20412x^2y^5$. (10) -18. (11) $-80a^4$.

(12) $\frac{280x^3b^4}{81}$. (13) $\frac{231x^6y^6}{16}$. (14) 70. (15) $\frac{663}{512}$.

(16) $2x^6+60x^4+120x^2+16$.

(18) The coefficients of $(1+x)^{14}$ are 1, 14, 91, 364, 1001, 2002, 3003, 3432, 3003, 2002, 1001, 364, 91, 14, 1.

(22) 0, 243, 1. (23) $\frac{21-n}{3n}$. (24) $1+\frac{18}{5}+\frac{18\,.\,17}{2\,.\,5^2}+\frac{18\,.\,17\,.\,16}{6\,.\,5^3}$.

(25) (i) the fifth; (ii) the fourth.

Ex. 97.

(1) $1+\frac{x}{4}-\frac{3x^2}{32}+\frac{7x^3}{128}$. (2) $1+\frac{2x}{3}-\frac{x^2}{9}+\frac{4x^3}{81}$. (3) $1-\frac{x}{5}-\frac{2x^2}{25}-\frac{6x^3}{125}$.

(4) $1-x+x^2-x^3$. (5) $1+x+x^2+x^3$. (6) $1+2x+\frac{5x^2}{2}+\frac{5x^3}{2}$.

(7) $1+\frac{4x}{3}+\frac{20x^2}{9}+\frac{320x^3}{81}$. (8) $\frac{1}{4}-\frac{x}{4}+\frac{3x^2}{16}-\frac{x^3}{8}$.

(9) $a^{-\frac{1}{3}}\left\{1+\frac{x}{3a}+\frac{2x^2}{9a^2}+\frac{14x^3}{81a^3}\right\}$. (10) $\frac{2^8\,.\,11\,.\,x^6}{3^8}=\cdot 43x^6$ about.

(11) $-\frac{2^{11}\,.\,13}{3^9}=-1\cdot 35\ldots$. (12) $462x^5$.

(13) $\frac{1\,.\,3\,.\,5\ldots\overline{2n-1}}{2^n\,.\,n\,!}x^n$. (14) $\frac{(-1)^{r-1}\,.\,5\,.\,1\,.\,3\,.\,7\ldots\overline{4r-13}}{4^{r-1}\,.\,r-1\,!}x^{r-1}$.

(15) $\frac{p(p+1)(p+2)\ldots(p+r-1)}{r\,!}x^r$.

(16) $\dfrac{p(p+q)(p+2q)\ldots(p+\overline{n-1}\,.\,q)x^n}{q^n\,.\,n\,!}$.

(17) 10·049. (18) 10·01. (19) 5·002.

(20) 9·950. (21) ·2011. (22) 1·995.

Ex. 98.

(1) 1, 1. (2) 7, 3. (3) 0, 0. (4) a, b. (5) 0, 0.

(6) c, d. (7) $2l$, $2m$. (8) c, b. (9) $\frac{3}{3}\frac{5}{3}$, $\frac{5}{3}\frac{}{3}$.

(10) $-3\frac{1}{2}$, $\frac{1}{1}\frac{4}{5}$. (11) 5, 3. (12) $\frac{1}{2}$, $\frac{1}{3}$. (13) 1, $1\frac{1}{2}$.

(14) -7, 2. (15) $4\frac{1}{2}$ mi./hr., 4 mi./hr. (16) 4 mi./hr., 5 mi./hr.

(17) $4\frac{1}{3}$, $-\frac{2}{3}$. (18) 0, 2. (19) 4, -1. (20) -2, 5.

(21) $a+b$, $a-b$. (22) l^2m, m^2l. (23) Pig £2. 10s., Sheep £3.

(24) 4 mi./hr., 12 mi./hr. (25) 10 mi./hr., 5 mi./hr.

(26) ±2, ±4. (27) -3, $\frac{1}{2}$, and 4 imag. roots.

(28) $\pm\sqrt{13}$, *i.e.* $\pm3{\cdot}6$, $\pm\sqrt{-12}$. (29) 4, 1. (30) $\frac{1}{9}$. (31) 1.

(32) 125, $-\dfrac{1331}{27}$. (33) 2, $\dfrac{1}{2}$, $\dfrac{1\pm\sqrt{-24}}{5}$.

(34) $10x^4-29x^3+30x^2-29x+10=0$. (35) -1, -1, -1, -1.

(36) 2, $\dfrac{1}{2}$, -3, $-\dfrac{1}{3}$. (37) 1, 1, $\dfrac{1\pm\sqrt{-3}}{2}$. (38) $\dfrac{3\pm\sqrt{-7}}{4}$, $\dfrac{-1\pm\sqrt{-15}}{4}$.

(39) $\dfrac{3\pm\sqrt{13}}{2}$ *i.e.* 3·303, $-$·303, $\dfrac{-1\pm\sqrt{5}}{2}$ *i.e.* ·618, $-1{\cdot}618$.

(40) ±1, $\dfrac{-7\pm\sqrt{53}}{2}$ *i.e.* $-7{\cdot}140$, ·140. (41) -1, $-\dfrac{1}{2}$, -2.

(42) 1, $\dfrac{-9\pm\sqrt{65}}{4}$ *i.e.* $-4{\cdot}265$, $-$·234.

(43) -1, 1, 1, $\dfrac{3\pm\sqrt{5}}{2}$ *i.e.* 2·618, ·618.

(44) -1, $-{\cdot}31\pm\sqrt{-{\cdot}90}$, ·81$\pm\sqrt{-{\cdot}34}$, $\left[x+\dfrac{1}{x}=\dfrac{1\pm\sqrt{5}}{2}\right]$.

(45) -1, $\dfrac{1-a\pm\sqrt{a^2-2a-3}}{2}$. (46) $\dfrac{a\pm\sqrt{a^2-4}}{2}$, $a\pm\sqrt{a^2-1}$.

(47) $(3, \frac{1}{2})$, $(-3, -\frac{1}{2})$, $(2\sqrt{2}, \frac{1}{2}\sqrt{2})$, $(-2\sqrt{2}, -\frac{1}{2}\sqrt{2})$.

(48) (2, 1), $(-2, -1)$, $\left(\dfrac{4\sqrt{15}}{15}, \dfrac{-9\sqrt{15}}{15}\right)$, $\left(-\dfrac{4\sqrt{15}}{15}, \dfrac{9\sqrt{15}}{15}\right)$.

(49) (3, 4), $(-3, -4)$, $\left(\dfrac{6\sqrt{13}}{13}, \dfrac{17\sqrt{13}}{13}\right)$, $\left(-\dfrac{6\sqrt{13}}{13}, \dfrac{-17\sqrt{13}}{13}\right)$.

(50) (2, 3), $(-2, -3)$, $\left(\dfrac{3\sqrt{74}}{37}, \dfrac{2\sqrt{74}}{37}\right)$, $\left(-\dfrac{3\sqrt{74}}{37}, \dfrac{-2\sqrt{74}}{37}\right)$.

(51) $\pm\sqrt{10}\pm 2\sqrt{2}$, $\pm\frac{1}{2}\sqrt{10}\mp\sqrt{2}$.

(52) $\pm\sqrt{\frac{28}{3}}$, $\pm\sqrt{\frac{20}{3}}$ *i.e.* (3·055, 2·582), (−3·055, −2·582).

(53) $(\pm\sqrt{2-\sqrt{5}}, \pm\sqrt{-2-\sqrt{5}})$, (2·058, ·487), (−2·058, −·487).

(54) $(4\frac{1}{2}, \frac{1}{2})$, $(-4\frac{1}{2}, -\frac{1}{2})$. (55) (5, 1), (−1, −5).

(56) (7, 4), (4, 7). (57) (3, 1), $(-\frac{13}{14}, \frac{39}{14})$.

(58) (±4, ±5), (±5, ±4), any signs.

(60) $u=7$, $v=6$, $x=6$, $y=1$, or $x=1$, $y=6$,
$u=-\frac{9}{5}$, $v=\frac{185}{25}$, $x=-\frac{3}{5}$, $y=-\frac{6}{5}$, or $x=-\frac{6}{5}$, $y=-\frac{3}{5}$.

(61) $u=7$, $v=10$, $x=5$, $y=2$, or $x=2$, $y=5$,
$u=-4$, $v=-1$, $x=-2+\sqrt{5}$, $y=-2-\sqrt{5}$, or $x=-2-\sqrt{5}$, $y=-2+\sqrt{5}$.

(62) $u=7$, $v=12$, $x=4$, $y=3$, or $x=3$, $y=4$,
$u=-10$, $v=29$, imag. roots: $u=0$, $v=\frac{3}{2}$, imag. roots.

(63) $u=7$, $v=6$, $x=6$, $y=1$, or $x=1$, $y=6$,
$u=-\frac{37}{5}$, $v=\frac{222}{25}$, $x=-1\cdot507$, $y=-5\cdot893$, or $x=-5\cdot893$, $y=-1\cdot507$.

(64) $-5\frac{1}{2}$. (65) 7. (66) $-\frac{1}{2}$. (69) (i) 3; (ii) 18.

(70) 4. (71) 5. (72) 13, 5. (73) 10.

(74) $\frac{4b-a}{3}$, $\frac{4a-b}{3}$. (75) $x^2=a^2+2b^2\pm 2b\sqrt{2a^2+b^2}$.

Ex. 99.

(4) (i) 2, −1, −1; (ii) 2, ·414, −2·414; (iii) ·853.

(6) (i) ·98; (ii) 357. (8) (i) 75·9; (ii) 56·25. (9) (i) 22·9; (ii) ·428.

(13) 1670. (14) 1·59 cents. (15) 5·2 in. (16) 1·34.

(17) Maximum is 16. (21) 1. (22) 1·39.

(23) −·25, −15·75. (26) 144 cub. in. (27) 28·9 cub. ft.

(29) 2·058, ·487, or −2·058, −·487. (30) 3·16, 2·45, or −3·16, −2·45.

(32) (i) Move origin to $(\alpha, 0)$; (ii) $\phi(\alpha)$.

Ex. 100.

(1) 136,000. (2) ·000,000,000,002,729. (3) 9932.

(4) ·2052. (5) 10,300,000. (6) 855,700.

(7) ·000,000,003,832. (8) 1599. (9) 9·076.

(10) ·8563. (11) ·1157. (12) 1·783.

(13) 3·739. (14) $x=$·949. (15) $x=-$·613, $y=$·387.

(16) 1030. (17) (α) $V=4\cdot832$; (β) $P=224\cdot5$.
(18) $3\cdot057\times10^9$. (19) $v_2=1\cdot831$. (20) $\cdot0631$.
(21) $3\cdot13$ or $-\cdot320$. (22) $x=\cdot7153$. (23) $8\cdot110$.
(24) $2\cdot204\times10^6$. (25) $2\cdot770$. (26) $472\cdot5$.
(27) $2\cdot72$. (28) $n=20$. (29) £1227, £103,400.
(30) £90,440, £5,292,000. (31) £743. (32) 128 years.
(33) £19,800. (34) £681. (35) £1263.

Ex. 101.

(1) -48. (3) $9x-14$, 2. (5) 0. (6) (i) 4; (ii) 13.
(8) $a^{12}+3a^{10}b^2+2a^9b^3+3a^8b^4+6a^7b^5+2a^6b^6+\ldots+b^{12}$.
(10) $1\cdot5$. (11) 9, 10. (12) 2.
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